Determination of the Chargino Mass Using Singular Value Decomposition (SVD) Method of a Complex Symmetric Matrix

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Abstract

The 2 x 2 complex mass matrix describes the mixing of the charged Gauginos and charged Higgsinos. Using the SVD method, the chargino masses are:

$$M^2_{x1,x2} = \frac{1}{2}\left[|\mu|^2 + |M_2|^2 + 2M_w^2 \mp \left[\left[|\mu|^2 + |M_2|^2 + 2M_w^2\right]^2 - 4|\mu|^2|M_2|^2 - 4M_w^4\, Sin\,\beta + 8M_w^2\, Sin\,2\beta\,\mathrm{Re}(\mu M_2)\right]^{1/2}\right]$$

where μ is the Higgsino mass parameter M_2 is the gaugino majorana mass $\beta = V_u/V_d$, V_d *and* V_u are the two Higgs vacuum expectation values. M_w is the mass of the w gauge boson.

1.0 Introduction

The standard model of particles describes matter as made up of particles, and these particles interact with each other by exchanging other particles associated with the fundamental forces[1]. The particles that make up matter are called fermions and the force carriers are called bosons.

Super-symmetry is a proposed space-time symmetry that relates fermions and bosons[2,3]. Each particle from one group is associated with a particle from the other known as its super particles. The spin of which differs by a half integer.

Each pair of super particles would share the same mass and internal quantum numbers in an unbroken super symmetry. The simplest realization of spontaneously broken super symmetry is the minimal super symmetric standard model.

In this theory, the Higgsinos and the gauginos mix with each other as a result of electroweak symmetry. The charged higgsinos $\left(H_u^+ \text{ and } H_d^-\right)$ and winos $\left(w^+ \text{ and } w^-\right)$ mix to form the mass eigenstates with charge ± 1 called charginos [4]. A gaugino is the hypothetical super partner of a gauge field.They are fermions. The winos are the super partners of the w-bosons. The Higgsinos are the super partners of the Higgs.

The chargino is a hypothetical particle which is a charged fermion predicted by super symmetry. Charginos are linear combinations of the charged winos and charged Higgsinos.

2.0 Theory and Methodology

2.1 Super Symmetry

Super symmetry, in Particle Physics, is a symmetry between fermions and bosons. Each particle from one group is associated with a particle from the other, known as its super-partners; the spin of which differ by an integer [2,3]. In a theory, with perfectly “unbroken” super-symmetry each pair of super partners would share the same mass and internal quantum numbers besides spin. However, since no super partners have been observed, if super-symmetry exists it must be a spontaneously broken symmetry so that super partners may differ in mass [5].

The mixing of the charger gauginos w and charged Higgsinos $\left(H_u^+ \; and \; H_d^-\right)$is described by a 2 x 2 complex mass matrix

$$M_c = \begin{bmatrix} M_2 & 1/\sqrt{2}\, gV_u \\ 1/\sqrt{2}\, gV_d & \mu \end{bmatrix} \text{---} 2.1$$

where M_2 is the wino mass term, V_u *and* V_d are the two Higgs vacuum expectation values, μ is the Higgsino mass parameter and g is the coupling constant. To determine, the physical chargino states and their masses, one must perform a singular value decomposition (SVD) [6,7] of the complex mass matrix Mc

2.2 Singular Value Decomposition

The singular value decomposition was originally developed by differential geometers, who wished to determine whether a real bilinear form could be made equal to another by independent orthogonal transformation of the two spaces it acts on. Eugenio Beltrami and Camille Jordan discovered independently, in 1873 and 1874 respectively, that the singular values of the bilinear forms, represented as a matrix, form a complete set of invariants for bilinear forms under orthogonal substitutions. James Joseph Sylvester also arrived at the singular value decomposition for real square matrices in 1889, apparently independently of both Beltrami and Jordan. Sylvester called the singular values the canonical multipliers of the matrix A. the fourth mathematician to discover the singular value decomposition independently is Autonne in 1915, who arrived at it via the polar decomposition. The first proof of the singular value decomposition for rectangular and complex matrices seems to be by Carl Eckart and Gale Young in (1936[8], they saw it as a generalization of the principal axis transformation for Hermitian matrices. In 1907, Erhard Schmidt defined an analog of singular values for integral operators, it seems he was unaware of the parallel work on singular values of finite matrices. This theory was further developed by Emile Picard in 1910, who is the first to call the members singular values. Practical methods for computing the Singular Value Decomposition (SVD) date back to Kogbetliantz in 1954, 1955 and Hestenes in 1958[9] resembling closely the Jacobi eigenvalue algorithm, which uses plane rotations or given rotations. However, these were replaced by the method of Gene Golub and William Kahan published in 1965[10] which uses householder transformations or reflections. In 1970, Golub and Christian Reinsch [11] published a variant of the Golub/Kuhan algorithm that is still the one most-used today.

In linear algebra, there are various methods to factorize matrices, singular value decomposition is one of that. SVD is a factorization of a matrix (real or complex). It has many useful applications in singular processing and statistics. SVD is divided into two sections: full SVD and reduced SVD. SVD is a method for identifying and ordering the dimensions along which data points exhibit the most variations. The SVD of an mxn real or complex matrix A can be broken down into the product of 3 matrices i.e.:

$$A_{mxn} = U_{mxm} D_{mxn} V^{*}_{nxm} \text{---} 2.2$$

Where

U = an orthogonal matrix

D = a diagonal matrix with non-negative real numbers and all its entries are singular value of A.

V* = transpose of an orthogonal matrix v(real or complex unitary matrix, if v is real).

Therefore the n column of v are called the left-singular vectors and the m columns of u are called right – singular vector of M, respectively.

2.2.1 Steps to Find The SVD

Step 1: Find the eigen-vector for AA^* and arrange them as a column vectors in a matrix ordered by the size of the corresponding eigenvalue, convert this matrix into an orthogonal matrix which we do by applying the Gram – Schmidt orthonormalization process to the column vectors.

Step 2: Calculate similarly (Step 1) for V^* using A^*A.

Step 3: For matrix D, we take the square roots in the non-zero eigenvalues and also populate the diagonal with these, putting the major in D_{11}, the next major in D_{22} and so forth until the smallest value results in D_{mm}.

Step 4: Substituting all the matrices in relation, $A_{mxn} = U_{mxm} D_{mxn} V^*_{mxn}$ and multiply them.

Step 5: Write the answer

Thus

$$U^* M_c V^{-1} = diag\left(M^+_{x1}, M^+_{x2}\right) \text{-------------------------------------} 2.3$$

Where U and V are unitary matrices and the right-hand side of the equation number 2.3 is the diagonal matrix elements and are the chargino masses.

The singular value decomposition (SVD) allows us to transform a symmetric matrix M to diagonal form using unitary matrices. For any complex (or real) nxn matrix M, unitary (or real orthogonal) matrices L and R exist such that; $L^T\ MR = M_D = diag\ (m_{1,}\ m_{2,}\ \cdots, m_n)$

where the M_K are real and non-negative. This is called the singular value decomposition of matrix M [12, 13].

3.0 Results

The complex mass matrix of the charged gauginos charged higgsinos is given in [14]

$$M_c = \begin{bmatrix} M_2 & 1/\sqrt{2}\, gV_u \\ 1/\sqrt{2}\, gV_d & \mu \end{bmatrix} \text{---------------------------------------3.1}$$

where M_2 is the wino mass term, V_u *and* V_d are the two Higgs vacuum expectation values, μ is the Higgsino mass parameter and g is the coupling constant.

To determine the physical chargino states and their masses, one must perform a singular value decomposition (SVD) of the complex mass matrix

$Mc : U^{*} McV^{-1} = diag\left(M_{x1}^{+}, M_{x2}^{+}\right)$--3.2

where U and V are unitary matrices and the right-hand side of the equation number 3.2 is the diagonal matrix of chargino masses.The singular value decomposition of the 2 x 2 complex symmetry matrix M_c is [15]

$$M_{x1}, M_{x2} = \frac{1/\sqrt{2}\left(\sqrt{2M_w^2 + |\mu|^2} + \sqrt{\left(2M_w^2 + |y|^2 + |M_2|^2\right)^2}\right.}{-4\left|M_w^2 \, Sin\, 2\beta - |\mu| |M_2|\right|}$$ ----------------------3.3

where $Sin\, \beta = V_u \Big/ \sqrt{V_u^2 + V_d^e}$

$\tan \beta = V_u / V_d$

The charginos are fermions and are electrically charged [16].The heavier chargino can decay through the neutral z-boson to the higher chargino. And both can decay through a charged w-boson to a neutralino[17].

4.0 Conclusion

The chargino masses obtained in this work are in agreement with the values in the literature. In the limit $M_z << |\mu \pm M_2|$, with real M_2 *and* μ, the chargino mass eigenstates consists of a wino-like $X_1^{\pm}$ is nearly degenerate with one of the neutroluos [4].

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