DEVELOPMENT OF SALTWATER INTRUSION MODEL IN COASTAL AQUIFERS

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ABSTRACT

Saltwater intrusion is a major challenge for the management of drinking water supply in the coastal regions. It is the most common and wide spread contamination problem in aquifers around the world due to increasing coastal population. The best means to understand, predict, and ameliorate saltwater intrusion, as well as to manage aquifers subject to saltwater intrusion involves mathematical modeling to simulate the effect of hydrogeophysical parameters on the mass flux of saltwater on coastal aquifers. A mathematical model was developed by applying some constraints on Darcy's and Fick's laws; it was used to simulate effect of geophysical parameters on mass flux of saltwater in coastal regions. The range of boundary value conditions were obtained empirically from a modeled experiment. Results showed that the mass flux of saltwater contaminant in a porous medium attenuates as a function of hydraulic conductivity and diffusion coefficient.

Key words: Seawater intrusion, groundwater, hydraulic gradient, hydraulic conductivity and diffusion coefficient.

INTRODUCTION

Population growth and continuous development require larger quantities of water, especially in the coastal regions where about 70% of the world population dwell. It is a great challenge to supply the required water, while the available water resources are nearly constant. This requires practical measures to protect the available resources from pollution, saltwater intrusion and other contaminants that deplete the current resources [1]. Saltwater intrusion is a major problem in coastal regions all over the world. In coastal areas the aquifers are in hydraulic contact with the sea. Under normal conditions the freshwater flows into the sea. However, over-pumping may result in inversion of the groundwater flow from the sea towards the inland causing saltwater contamination. Also the rise in sea level will accelerate the saltwater intrusion by virtue of increase in piezometric height. Salinization of groundwater is considered a special category of pollution that threatens groundwater resources, because mixing a small quantity (2 percent) of saltwater with groundwater makes freshwater unsuitable and can result in abandonment of freshwater supply. Therefore, saltwater intrusion should be prevented or at least controlled to protect groundwater resources.

A number of different measures have been used to control seawater intrusion and to protect the groundwater resources. The main principle of protection is to increase the volume of fresh groundwater and reduce the volume of saltwater. Todd D.K discussed various means of preventing saltwater from contaminating groundwater sources including: (1) reduction of the abstraction rates, (2) relocation of abstraction wells, (3) subsurface barriers, (4) natural recharge, (5) artificial recharge, (6) abstraction of saline water, and (7) combination of injection and abstraction systems [2]. Extensive research has been carried out to investigate saltwater intrusion in coastal aquifers. However, only few models have been developed to study the control of saltwater intrusion. These models use one or more of the previous measures to study the control of saltwater intrusion. The reduction of abstraction rates aims to reduce the pumping rates and use other water resources [3]. The relocation of abstraction well aims to move the wells further inland [4]. Subsurface barriers aim to prevent the inflow of seawater into the basin [5]. Natural recharge aims to recharge aquifers with additional surface water [6]. Artificial recharge aims to increase the groundwater levels, using surface spread for unconfined aquifers and recharge wells for confined aquifers. The sources of water for injection may be surface water, groundwater, treated wastewater or desalinated water [7]. The abstraction of saline water aims to reduce the volume of saltwater by extracting brackish water from the aquifer [8]. The combination of injection of freshwater and extraction of saline water can reduce the volume of saltwater and increase the volume of freshwater [9].

The main objectives of this work are to develop a mathematical model using empirical input as boundary condition value and validate the model by comparing the result with a modeled laboratory experiment.

THEORY

Subsurface water flow is governed by Darcy's law and the transport of contaminant in coastal aquifers such as saltwater intrusion is based on Fick's laws of diffusion. The properties of the coastal aquifers such as hydraulic conductivity, effective porosity and hydraulic gradient affect saltwater intrusion into fresh groundwater aquifer. These properties of aquifer contribute to its ability to reduce the severity of groundwater contamination which is known as soil attenuation or soil filtration.

According to Darcy's law, the volumetric flow rate per unit area (volume flux) is directly proportional to the hydraulic gradient [10, 11]

$$q = - k\frac{\rho}{\mu}g\frac{dh}{dl} \qquad (1)$$

where k is the intrinsic permeability of the porous media, μ is dynamic viscosity of the fluid, ρ is density of the fluid, g is acceleration due to gravity and $\frac{dh}{dl}$ is the hydraulic gradient.

The term $k\frac{\rho}{\mu}g$ in equation 1 is hydraulic conductivity which is denoted by K.

According to first Fick's law of diffusion, the mass flux in kgm^{-2}s^{-1} is the amount of mass of saltwater passing through a unit area per unit time and this is directly proportional to the gradient of concentration [12].

$$J \propto -\frac{dc}{dx} \qquad (2)$$

$$J = -D^{*}\frac{dc}{dx} \qquad (3)$$

Where J is mass flux of substance (Kg/m^{2}s), D* is coefficient of diffusion and $\frac{dc}{dx}$ is gradient of concentration in space.

According to one dimensional second Fick’s law of diffusion, the rate of change of concentration with time is given as

$$\frac{dc(x,t)}{dt} \propto \frac{d^2c(x,t)}{dx^2}$$

$$\frac{dc(x,t)}{dt} = D^* \frac{d^2c(x,t)}{dx^2} \qquad (4)$$

Where c is the concentration of saltwater in porous medium, t is the time of diffusion and x is the distance (length) of diffusion.

Introducing the element of time to equation 3 we have:

$$J = -\frac{D^*}{V}\frac{dc}{dt} \qquad (5)$$

Also the concentration of saltwater as exponential function of time t can be express as

$$C = C_o \exp\left(-\lambda_t t\right) \qquad (6)$$

The concentration of saltwater as exponential function of distance x can be express as

$$C = C_o \exp\left(-\lambda_x x\right) \qquad (7)$$

where λ_t and λ_x are coefficient of attenuation and filtration, respectively.

Differentiate equation 6 and 7 with respect to time t and x (twice) gives, respectively

$$\frac{dc}{dt} = -\lambda_t C_o \exp\left(-\lambda_t t\right) \qquad (8)$$

$$\frac{d^2c}{dx^2} = \lambda_x^2 C_o \exp(-\lambda_x x) \qquad (9)$$

Substitute equations 8 and 9 into equation 4 gives

$$- \lambda_t C_o \exp(-\lambda_t t) = \lambda_x^2 D^* C_o \exp(-\lambda_x x) \tag{10}$$

Natural logarithm of equations 6 and 7 were determined equate together to give

$$-\lambda_t t = -\lambda_x x$$

$$\lambda_t = \lambda_x \frac{x}{t} \qquad \text{(i.e } \frac{x}{t} = q \text{)}$$

Thus, $$\lambda_t = \lambda_x q \tag{11}$$

Substitute equation 11 into 10,

$$-\lambda_x q C_o \exp(-\lambda_x qt) = D^* \lambda_x^2 C_o \exp(-\lambda_x x)$$

$$-q C_o \exp(-\lambda_x x) = D^* \lambda_x C_o \exp(-\lambda_x x)$$

$$\lambda_x = -\frac{q}{D^*} \tag{12}$$

Combining equations 5 and 8

$$J = \frac{D^*}{q} \times \lambda_t C_o \exp(-\lambda_t t)$$

$$J = \frac{D^*}{\lambda_x D^*} \times \lambda_t C_o \exp(-\lambda_t t)$$

$$J = -q C_o \exp(-\lambda_t t)$$

$$J = -q C_o \exp(-\lambda_x q t)$$

$$J = -q C_o \exp(-\lambda_x x)$$

$$J = K C_o \exp\left(-\frac{q}{D^*} x\right)\frac{dh}{dl} \tag{13}$$

METHODOLOGY

The model derived in equation 13 describes the effect of hydraulic conductivity K, volume flux q and hydraulic gradient *i* ($\frac{dh}{dl}$) on mass flux of saltwater contaminant in a porous medium. Equation 13 was used to simulate the mass flux of saltwater contaminant by introducing the empirical properties of the media obtained from the experimental results in Table 1 using MATLAB software. Considering the distance of flow *X*, mass flux of contaminant was considered at interval $\Delta X = 0.05m$ from the reference point where it was assumed that the hydraulic gradient *i* vary at the rate of multiple of *i=1.875* for every increment $\Delta X.$ If for every ΔX it can be assumed that hydraulic gradient is $2^n i$ where n= 0,1,…4. If four sites of varying *i* was considered; at each of this location the corresponding value of mass flux J can be determined using the model. The results obtained were presented in Tables 2 for composite arrangement comprising the whole samples ABCD and the same pattern was used to obtain results for homogeneous samples A, B, C and D.

Table 1: Values of porosities, hydraulic conductivities and diffusion coefficients for composite arrangement ABCD and homogeneous samples A, B, C and D.

Samples	**Porosity (Ø)**	**Hydraulic Conductivity** **K× 10^{-5} (ms^{-1})**	**Diffusion Coefficient** **D× 10^{-5} (m^{2}s^{-1})**

ABCD	0.235±0.003	0.0081	0.05
A	0.250±0.010	0.42	0.30
B	0.300±0.002	3.13	0.40
C	0.330±0.001	4.45	0.50
D	0.420±0.010	7.05	0.61

Table 2: Numerical values of mass flux J generated for composite arrangement comprising the whole samples ABCD.

X(m)	n=0 $J_1(kgm^{-2} s^{-1})$ $\times 10^{-3}$	n=1 $J_2(kgm^{-2} s^{-1})$ $\times 10^{-3}$	n=2 $J_3(kgm^{-2} s^{-1})$ $\times 10^{-3}$	n=3 $J_4(kgm^{-2} s^{-1})$ $\times 10^{-3}$	n=4 $J_5(kgm^{-2} s^{-1})$ $\times 10^{-3}$
0	1.558	3.116	6.232	12.464	24.928
0.05	1.334	2.230	3.393	3.695	2.190
0.10	1.115	1.697	1.847	1.095	0.193
0.15	0.988	1.252	1.006	0.325	0.017

0.20	0.848	0.924	0.548	0.096	0.002
0.25	0.729	0.682	0.298	0.029	0.000
0.30	0.626	0.503	0.162	0.009	0.000
0.35	0.538	0.371	0.088	0.003	0.000
0.40	0.462	0.274	0.048	0.001	0.000
0.45	0.397	0.202	0.026	0.000	0.000
0.50	0.341	0.149	0.014	0.000	0.000

Table 3: Values of mass flux, J at distance 0.5m for both homogeneous and composite arrangement ABCD media.

Samples	n=0 J_1(kgm^{-2}s^{-1}) ×10^{-3}	n=1 J_2(kgm^{-2}s^{-1}) ×10^{-3}	n=2 J_3(kgm^{-2}s^{-1}) ×10^{-3}	n=3 J_4(kgm^{-2}s^{-1}) ×10^{-3}	n=4 J_5(kgm^{-2}s^{-1}) ×10^{-3}
ABCD	0.260	0.325	0.525	1.525	3.125

A	1.350	2.225	2.325	7.525	16.350
B	5.975	14.930	30.350	60.800	121.100
C	9.425	21.700	44.250	88.150	175.800
D	15.900	34.500	68.300	136.050	272.500

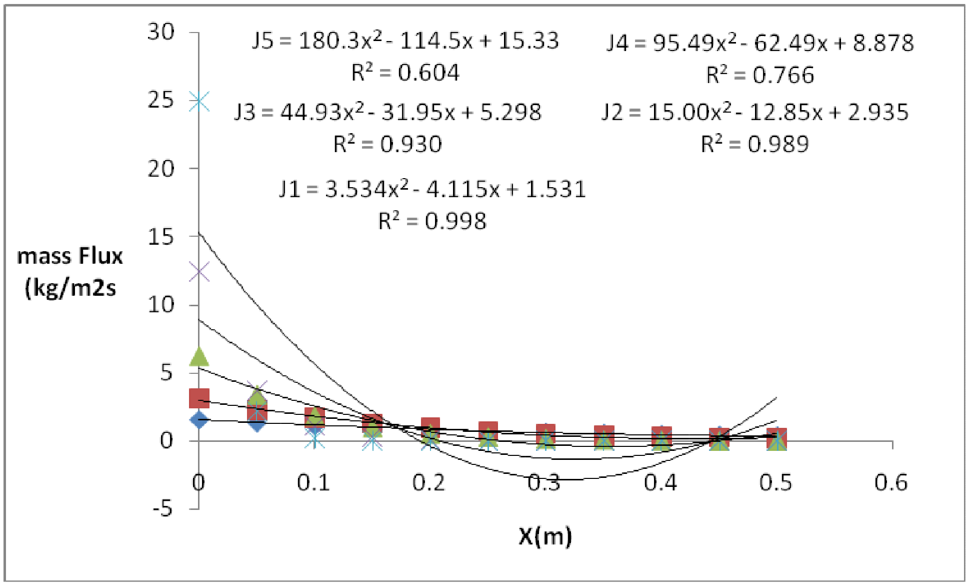

Fig. 1: Mass flux J against diffused length X (m) for heterogeneous ABCD

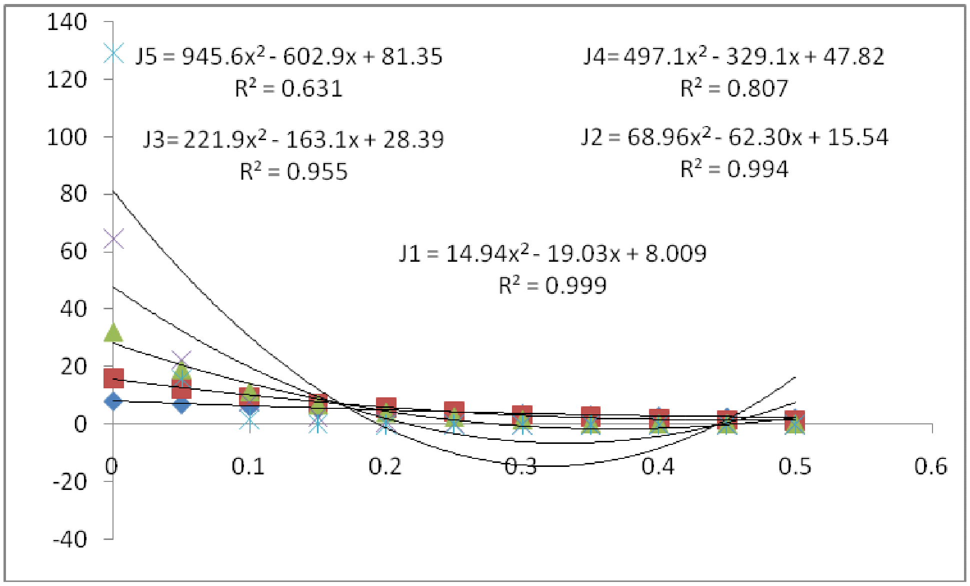

Fig. 2: Mass flux J against diffused length X (m) for heterogeneous A

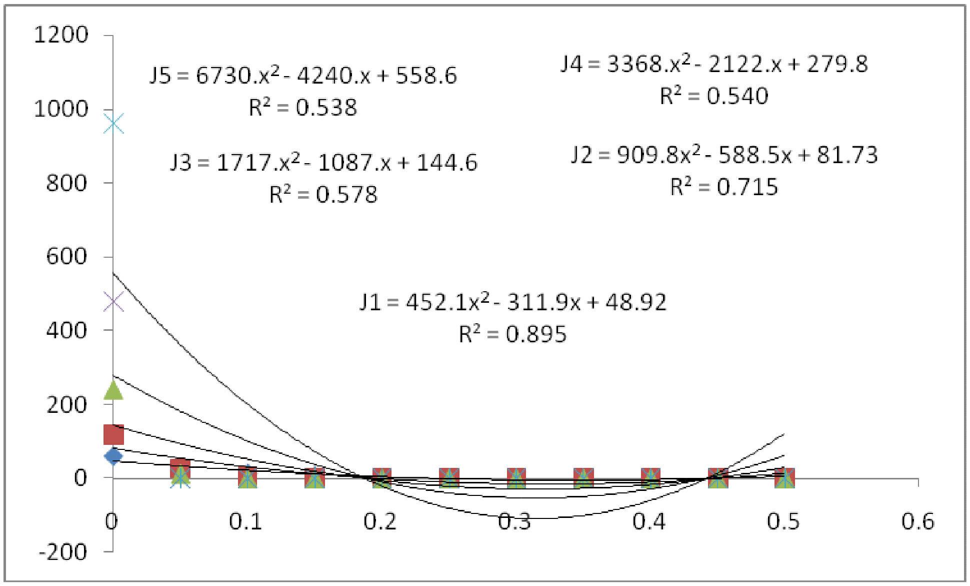

Fig. 3: Mass flux J against diffused length X (m) for heterogeneous B

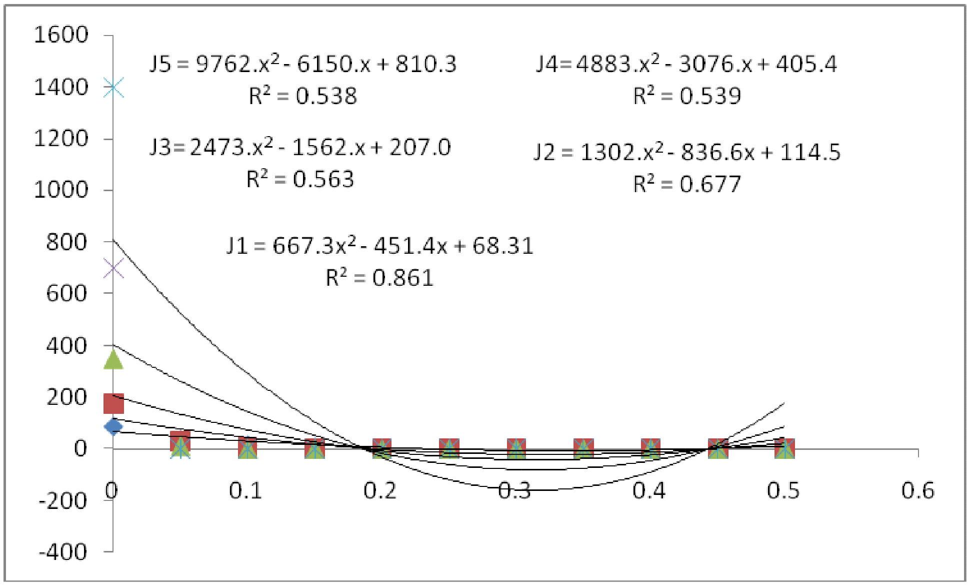

Fig. 4: Mass flux J against diffused length X (m) for heterogeneous C

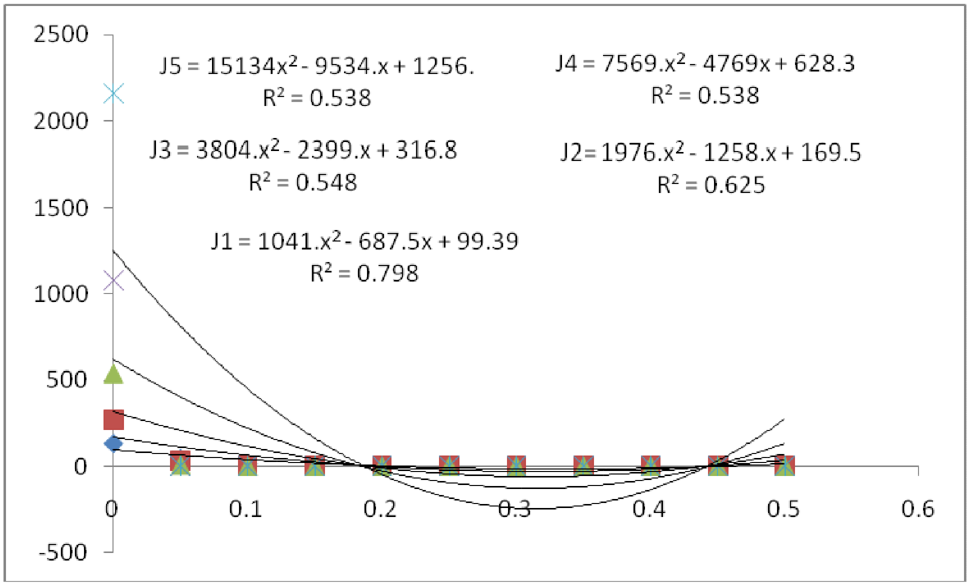

Fig. 5: Mass flux J against diffused length X (m) for heterogeneous D

RESULT, DISCUSION AND CONCLUSION

Mass flux of saltwater contaminant simulated in composite arrangement ABCD was presented in Tables 2. Mass flux of contaminant simulated for samples A, B, C and D follow the same pattern. Plots of mass flux J against flow length X for various hydraulic gradients for composite arrangement ABCD and homogeneous samples A, B, C, and D were shown in Figure 1 to Figure 5, respectively. These plots were polynomial of order 2 and the mass flux decay exponentially with flow length X. It can be used to determine the mass flux contaminant at any given of flow length from the interface of saltwater and freshwater. The mass flux, J at the flow length 0.5m for both composite arrangement ABCD and homogeneous samples A, B, C and D for various hydraulic gradients were obtained from the equations display in Figure 1 to Figure 5 and presented in Table 3. This indicates that the mass flux of contaminant is attenuated with decrease in hydraulic conductivity and diffusion coefficient which is achieved conveniently when the porosity of the medium is low as well as hydraulic gradient between freshwater and saltwater columns. Therefore, composite arrangement of the samples as ABCD controls saltwater intrusion better than homogeneous samples A, B, C and D. The developed model predicted and gave the best medium that controls saltwater intrusion in coastal regions.

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