

On the Comparative Study of Initial Basic Feasible Solution Methods for Transportation Problem using Temporary Ordered Routing Algorithm as a Validation Tool

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ABSTRACT

This paper presents a comparative study of Initial Feasible Solution Methods for Transportation Problem using Temporary Ordered Routing Algorithm (TORA) as a Validation Tool. To achieve this, various methods of finding initial feasible solutions including Northwest Corner Method, Least Cost Method, and Vogel Approximation Method were considered. These methods were compared on a balanced transportation model to provide the basis of finding initial solutions. A comparison of results (Initial feasible solutions & Optimal solutions) obtained from these various methods were presented along with the number of iterations required for the solutions. Findings from the results of several runs of the tested problem reveal that, the North West Corner Method, although fast in obtaining initial solutions (and theoretically very simply to implement), is not very efficient and accurate enough in finding the initial feasible solutions of this type of transportation problem. Also, the objective value for the transportation cost of the initial basic feasible solutions obtained through the Least Cost Method and Vogel's Approximation Method were in good agreement with the optimal solution or exactness. Thus, Vogel Approximation Method, (being an improved of Least Cost Method) is effective for the solution of this type of problem.

Key words: Transportation problem, Linear Programming, Northwest Corner Method,

1. INTRODUCTION

The Transportation Problem refers to a special class of Linear Programming where the objective is to provide optimum solution to problems whose product are to be transported or distributed from where the total available from each source and the total requirement at each destination are known. In other words, the problem is, how should transport be made so as to optimize the given objective [4].

The general method of solution for transportation problems is to first establish the problem matrix with each origin as a row and each destination as a column. Also, an initial solution must be obtained that satisfies all constraints on the matrix model. A matrix with m origins and n destinations will have $m + n - 1$ quantities of goods shipped from origin i to destination j . A test is performed on the matrix to see if there exist solutions that will provide a lower total cost to the company. If one exists, a new solution is found and this new solution is in turn tested for a possible better solution. Each solution which is tested, and the new solution formed is called an *iteration*. On a large problem, thousands of iterations may be required before an optimal solution can be obtained [2, 6].

[1,5] have developed different types of transportation models. Extension of the transportation model and methods have been subsequently studied [3]. Also, a systematic study and new approach was suggested to find the optimal solution of the transportation problem [7]. However, the aim of this paper is to compare various feasible solution techniques for solving transportation problem in linear programming and then validate the iterative results from the comparative simulation of these methods using Temporary Ordered Routing Algorithm (TORA).

2. TRANSPORTATION PROBLEMS AND DESCRIPTION OF METHODS

In this section, we present the transportation problem in linear programming and then give the general description of the various solution methods in use.

2.1 Formulation of the Transportation Problem

Given m origins and n destinations, the transportation problem can be formulated as the following linear programming problem model:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (2.1)$$

$$\text{Subject to: } \sum_{j=1}^n x_{ij} = a_i, \text{ for all } i = 1, 2, \dots, m \quad (2.2)$$

$$\sum_{i=1}^m x_{ij} = b_j, \text{ for all } j = 1, 2, \dots, n \quad (2.3)$$

$$\text{And } x_{ij} \geq 0, \text{ for all } i \text{ and } j. \quad (2.4)$$

Where x_{ij} is the amount of units of shipped from origin i to destination j and c_{ij} is the cost of shipping one unit from origin i to destination j . The amount of supply at origin is a_i and the amount of destination is b_j . The objective is to determine the unknown x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand constraints.

2.1.1 Balanced Transportation Problem

If the sum (total availability from origin) of the supplies of all the sources is equal to the sum of the demands of all the destinations, then the problem is termed as a Balanced Transportation Problem. Here,

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \quad (2.5)$$

2.1.2 Unbalanced Transportation Problem

If the sum (total availability from origin) of the supplies of all the sources is not equal to the sum of the demands of all the destinations, then the problem is termed as an Unbalanced Transportation Problem. Here,

$$\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j \quad (2.6)$$

An unbalanced Transportation problem can be modified to a balanced one by introducing a

dummy sink (destination) if $\sum_{i=1}^m a_i > \sum_{j=1}^n b_j$ and a dummy source if $\sum_{i=1}^m a_i < \sum_{j=1}^n b_j$.

The inflow from the source to a dummy sink represents the surplus at the source. Similarly, the flow from the dummy source to a sink represents the unfulfilled demand at the sink. The costs of transporting a unit from a dummy source to a dummy sink are assumed to be zero. The resulting problem is now balanced and can be solved.

2.2 North-West Corner Method (NWCM)

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North-West corner (i.e. top left corner). The method starts at the northwest-corner cell (route). The major advantage of the north-west corner rule method is that it is very simple and easy to apply. Its major disadvantage, however, is that it is not sensitive to costs and consequently yields poor initial solutions. The northwest corner method is by far the simplest method for obtaining the initial solution. This method gets its name because it starts with the cell in the upper left hand corner, x_{11} , and allocates the maximum feasible amount available at this point. In other words, the minimum between the amount available at origin one and the amount required at destination one. Move one cell to the right if there is any remaining amount at the origin, otherwise move one cell

down. If it is impossible to do either, then the solution has been formed. After each move allocate the maximum feasible amount and move again.

2.2.1 Steps:

- i. Select the north west (upper-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, i.e., $\min(s_1, d_1)$.
- ii. Adjust the supply and demand numbers in the respective rows and columns allocation.
- iii. If the supply for the first row is exhausted then move horizontally to the next cell in the second row.
- iv. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.
- v. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.
- vi. Continue the procedure until the total available quantity is fully allocated to the cells required.

2.3 Least-Cost Method (LCM)

Least-Cost method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation.

Matrix minimum method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation. The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted

accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the northwest –corner method. Next, look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out.

2.3.1 Steps

- i. Identify the box having minimum unit transportation cost (c_{ij})
- ii. If there are two or more minimum costs, select the row and the column corresponding to the lower numbered row.
- iii. If they appear in the same row, select the lower numbered column.
- iv. Choose the value of the corresponding x_{ij} as much as possible to the capacity and requirement constraints.
- v. If demand is satisfied, delete the column.
- vi. If supply is exhausted, delete the row.
- vii. Repeat steps 1-6 until all restrictions are satisfied.

2.4 Vogel's Approximation Method (VAM)

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The opportunity cost for a given supply row or demand column is defined as the difference between the lowest cost and the next lowest cost alternative. This method is preferred over the methods discussed above because it generally yields, an optimum, or close to optimum, starting solutions. Consequently, if we use the initial solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced. The steps involved in determining an initial solution using VAM are as follows:

2.4.1 Summary of Steps:

- i. Determine the penalty cost for each row and column.
- ii. Select the row or column with the highest penalty cost.
- iii. 3. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
- iv. Repeat steps i, ii, and iii until all rim requirements have been met

3. NUMERICAL RESULTS AND DISCUSSION

In this section, we illustrate the comparative study by problem statements. Data from these examples are secondary. All the results are implemented and calculated using TORA.

3.1 Numerical Problem: We consider the Numerical problem presented in Table 1 below:

We have used three methods to find an initial basic feasible solution for the balanced transportation problem. The problem was developed by Jim Caldwell and Douglass K.S.

Table 1: Transportation Cost per desk for Numerical Problem 1

<i>From</i>	To	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Supply</i>
1		8	14	12	17	a1=20
2		11	9	15	13	a2=10
3		12	19	10	6	a3=10
4		12	5	13	8	a4=5
Demand		b1=5	b2=20	b3=10	b4=10	

Source:Mathematical Modeling, Case Studies & Projects (p.233-234), by Jim C. & Douglas K.S, 2009, Kluwer Academic Publishers, New Delhi, India.

3.1 Model formulation

The problem is solved by first expressing it mathematically as a Linear Programming problem.

Let X_{ij} = Decision variables i.e units shipped from Breweries i to distribution centre (Hotels) j

Using the shipping cost data of Table 4.1, the transportation cost (in hundreds of naira) is written

as:

$$Z = 8x_{11} + 14x_{12} + 12x_{13} + 17x_{14} + 11x_{21} + 9x_{22} + 15x_{23} + 13x_{24} \\ + 12x_{31} + 19x_{32} + 10x_{33} + 6x_{34} + 12x_{41} + 5x_{42} + 13x_{43} + 8x_{44}$$

Subject to: Production constraints for breweries 1, 2, 3, 4 are:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 20$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 10$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 10$$

$$x_{41} + x_{42} + x_{43} + x_{44} \leq 5$$

Demand constraints for hotels A, B, C, D are:

$$x_{11} + x_{21} + x_{31} + x_{41} \geq 5$$

$$x_{12} + x_{22} + x_{32} + x_{42} \geq 20$$

$$x_{13} + x_{23} + x_{33} + x_{43} \geq 10$$

$$x_{14} + x_{24} + x_{34} + x_{44} \geq 10$$

All quantities transported must be nonnegative. Thus

$$x_{ij} \geq 0, \quad i, j = 1, 2, 3, 4.$$

3.2 Computation:

Three methods have been used here to find basic feasible solution of the above problem and these are presented in turn.

Computation using North West Corner Methods:

Steps:

(i) Comparing a_1 and b_1 . Since $b_1 < a_1$, we allocate 5 to x_{11} or (1,A). This exhaust the demand at b_1 and leave 15 units as unsatisfied supply at supply A.

(ii) Move to cell (1, B). Comparing a_1 and b_2 , we allocate 15 to x_{12} or (1, B)

(iii) Move to cell (2, B). Compare a_2 and b_2 , we allocate 5 to (2, B)

(iv) Move to cell (2, C). Compare a_2 and b_3 . We allocate 5 to cell (2, C)

(v) Move to cell (3, C). Compare a_3 and b_3 . Allocate 5 to cell (2, C)

(vi) Move to cell (3, D). Compare a_3 and b_4 , allocate 5 to cell (3, D)

(v) Move to cell 4, D. Compare a_4 and b_4 and continuing in this fashion gives:

Table 2: Solution to Numerical Problem Using North West Corner Method.

	To	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Supply</i>
<i>From</i>						
1		8 [5]	14 [15]	12	17	20
2		11	9 [5]	15 [5]	13	10
3		12	19	10 [5]	6 [5]	10
4		12	5	13	8 [5]	5
Demand		5	20	10	10	

Therefore, the total transportation cost (Initial Basic Feasible Solution) determined by the Northwest corner method is:

$$\text{Minimize } Z = (5 \times 8 + 15 \times 14 + 5 \times 9 + 5 \times 15 + 5 \times 10 + 6 \times 5 + 8 \times 5)$$

$$= 40 + 210 + 45 + 75 + 50 + 30 + 40 = 490$$

3.2.1 Validation of Numerical Problem using TORA:

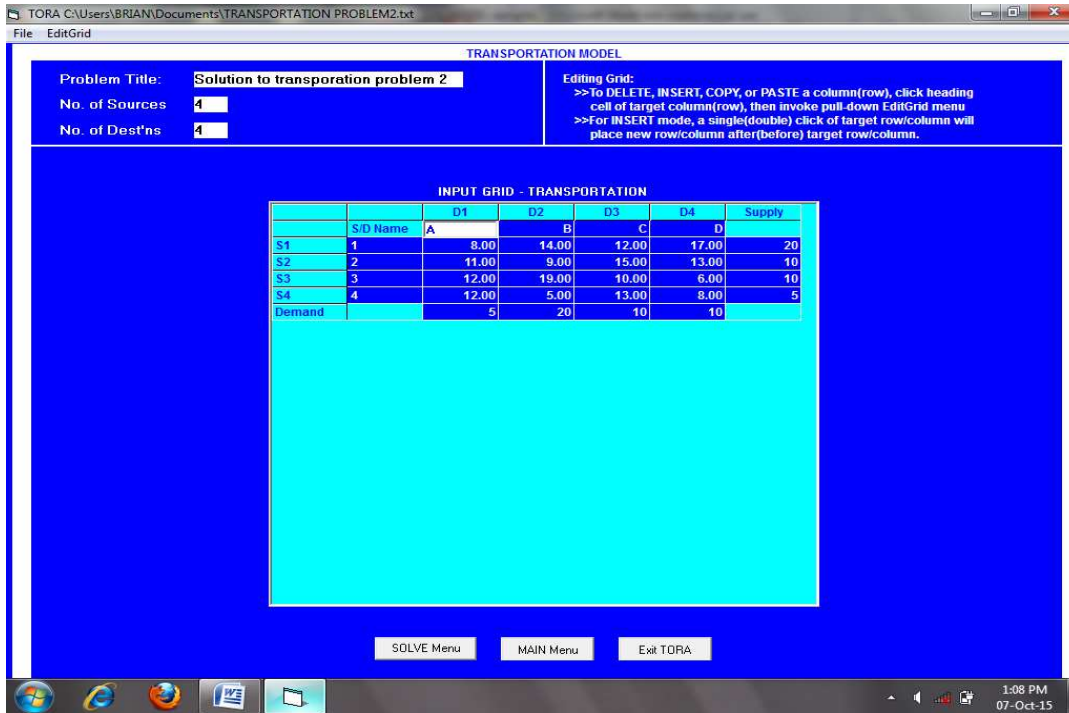


Figure 1. Original Data – Problem 1

3.2.2 Iteration 1: Objective Value = 490.00

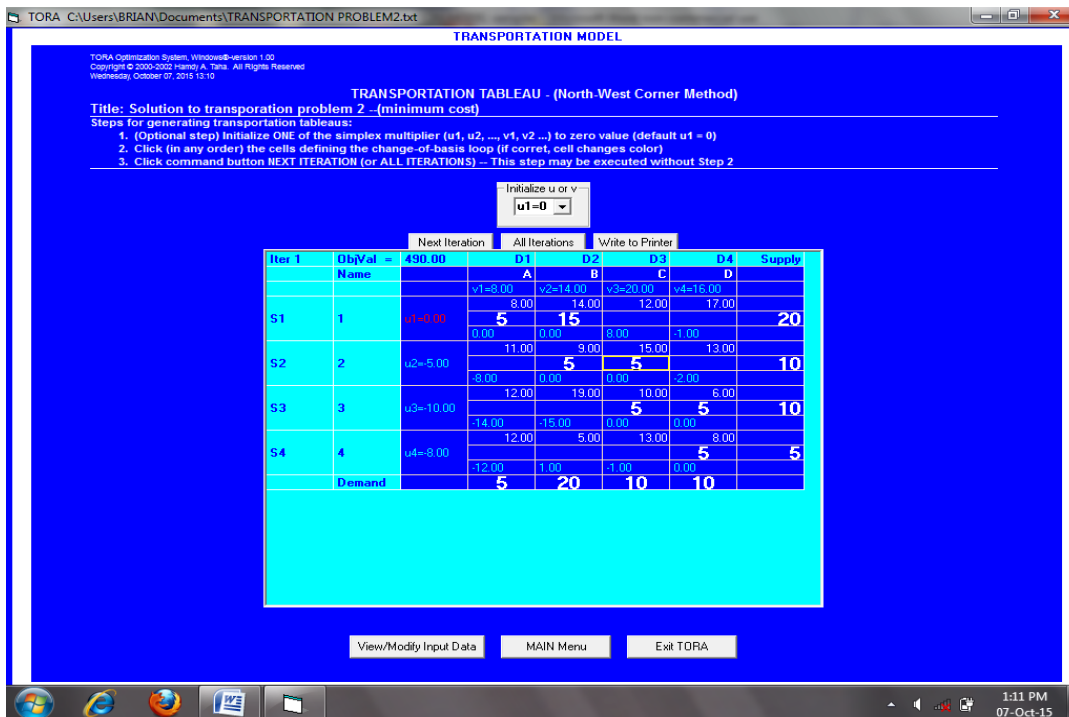


Figure 2. Northwest – corner method results

3.2.3 Iteration 2: Objective Value =450.00

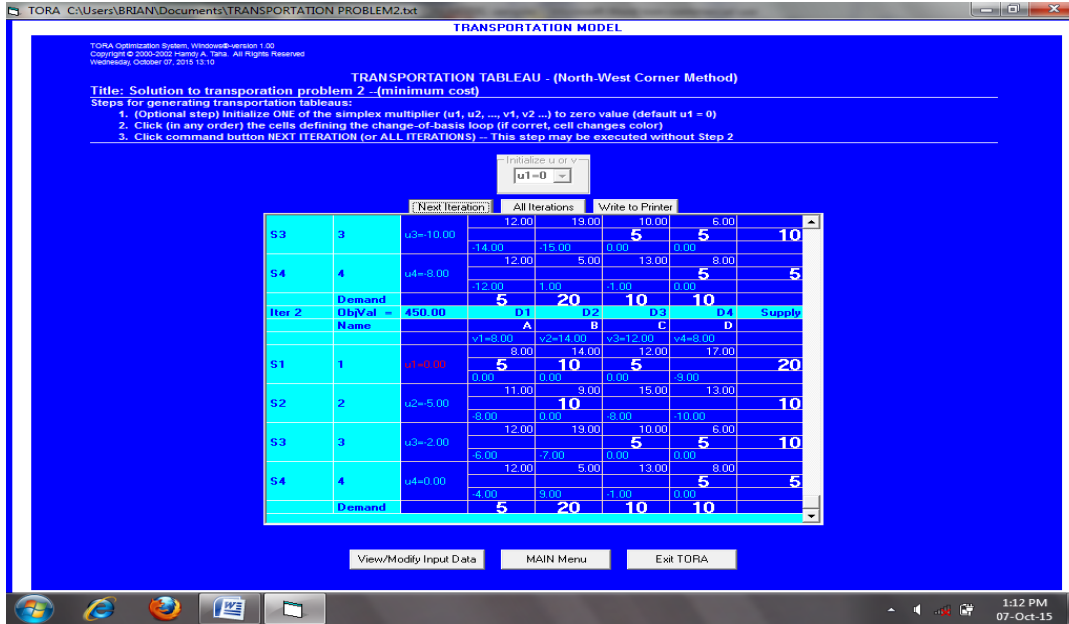


Figure 3. Northwest – corner, Iteration 2

3.3.4 Iteration 3: Objective Value =405.00 (OPTIMAL)

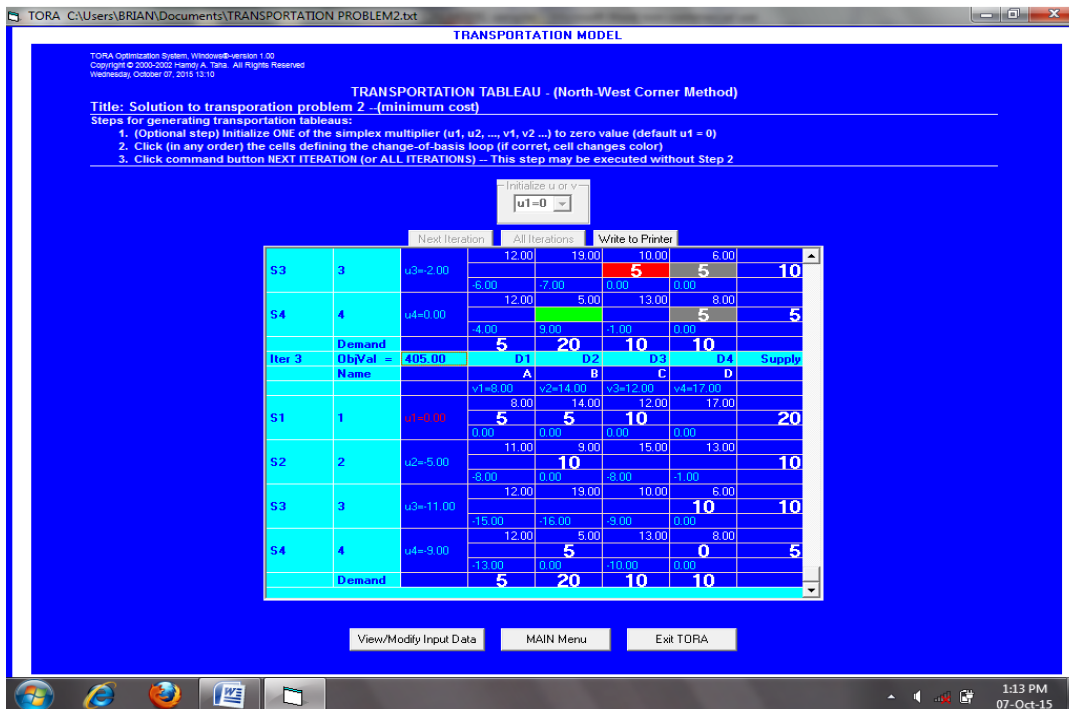


Figure 4. Northwest – corner, Iteration 3

3.3 Computation (Least Cost Method)

Using Least Cost Method (LCM) the final solution is presented in Table 3.

Table 3: Solution to Numerical Problem 1 using Least Cost Method.

From \ To	A	B	C	D	Supply
1	8 [5]	14 [5]	12 [10]	17[0]	20
2	11	9 [10]	15	13	10
3	12	19	10	6 [10]	10
4	12	5 [5]	13	8	5
Demand	5	20	10	10	

Therefore, the total transportation cost (Initial Basic Feasible Solution) determined by the Least Cost method is:

$$\begin{aligned} \text{Minimize } Z &= 5 \times 8 + 5 \times 14 + 10 \times 12 + 0 \times 17 + 10 \times 9 + 10 \times 6 + 5 \times 5 \\ &= 40 + 70 + 120 + 0 + 90 + 60 + 25 \\ &= 405 \end{aligned}$$

3.3.1 Validation of Least Cost Method using TORA

3.3.2 Iteration 1: Objective Value = 405.00 (OPTIMAL)

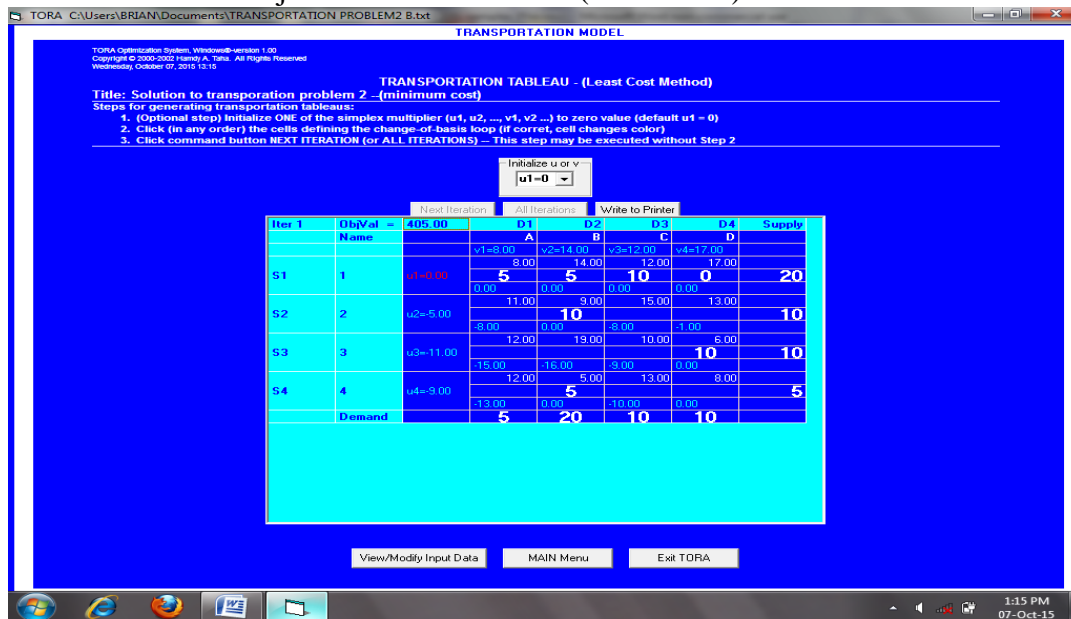


Figure 5. Least cost – method, Iteration 1

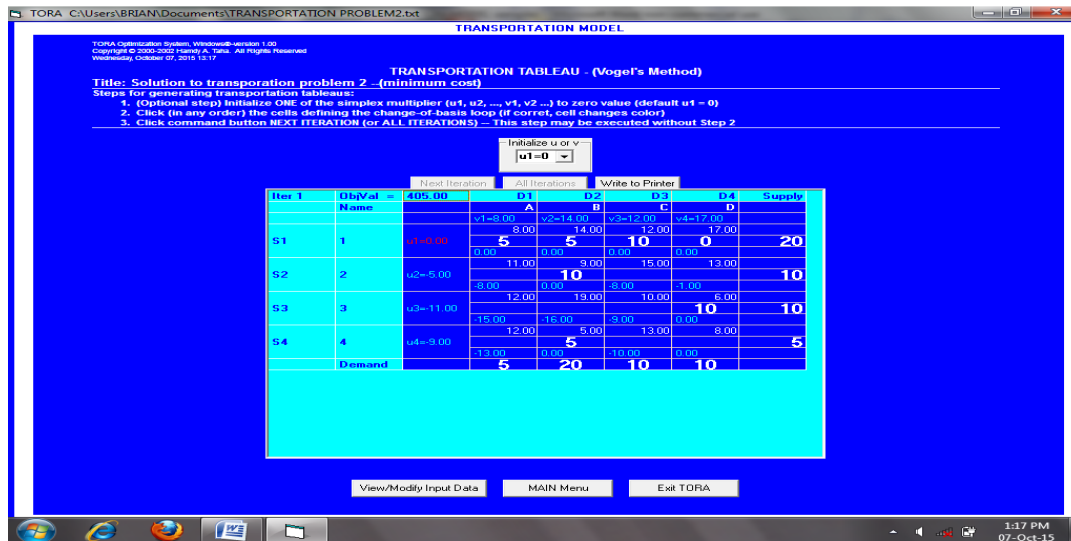


Figure 6. Vogel's approximation method



Figure 7. Transportation Model Output Summary to the Numerical Problem

Table 5: Comparison of different methods for finding initial basic solution to Numerical Problem with TORA as Validation tool

Methods Used	Initial Basic Feasible Solutions	Optimal Solutions	Iterations to reach Optimal Solutions
NWC method	490	405	3
Least cost method	405	405	1
Vogel approximation	405	405	1

3.5 Discussion of Results

The transportation model tested above had 4 origins and 4 destinations. Since the total supply at the origin is equal to the total demand, the problem is balanced. The number of iterations required to establish an optimal solution ranged from 3 for the Northwest corner method to 1 for Least-cost method and 1 for Vogel’s approximation method. The objective function (initial basic feasible solution method) for the NWC method is 490 while the objective function for the least cost method is 405 and 405 for the Vogel’s approximation method. Table 5 contains a summary of the comparative study for the transportation model problems.

Furthermore, the transportation model tested was solved and validated with TORA. The results from both the computation and that of TORA yielded the same values, in terms of the optimal solution obtained. The computer solution shows the minimum total transportation cost is 405. The value for the decision variables shows the optimal amount of beer to be shipped over each route.

To minimize the transportation cost, the following shipments should be made:

Ship 5 units of beer from Brewery 1 to Hotel A

Ship 5 units of beer from Brewery 1 to Hotel B

Ship 10 units of beer from Brewery 1 to Hotel C

Ship 0 units of beer from Brewery 1 to Hotel D

Ship 10 units of beer from Brewery 2 to Hotel B

Ship 10 units of beer from Brewery 3 to Hotel D

Ship 5 units of beer from Brewery 4 to Hotel B

4.CONCLUSION

Findings from the results of several runs of the tested problem reveal that, the North West Corner Method, although fast in obtaining initial solutions (and theoretically very simply to implement), is not very efficient and accurate enough in finding the initial feasible solutions of this type of transportation problem. This is because, the method is not sensitive to costs and consequently yields poor initial solutions. The objective value for the transportation cost of the initial basic feasible solutions obtained through the Least Cost Method and Vogel's Approximation Method were in good agreement with the optimal solution or exactness. Thus, Vogel's Approximation Method, (being an improved of Least Cost Method) is ineffective for the solution of this type of problem.

REFERENCES

- [1] Chaudhuri, A “A. Comparative study of Transportation Problem” *Journal of Mathematics and Computer Science*, (2011), 256-267.
- [2] Gupta P. K. & Hira D.S. Operations Research, 6th Edition, S.Chand & Company Ltd, New Delhi, India. (2011), 233-329.
- [3] Hakim M.A. An Alternative Method to Find Initial Basic Feasible Solution of a Transportation Problem, *Annals of Pure and Applied Mathematics*, Vol. 1, No. 2, (2012), 203-209.
- [4] Jim C. and Douglass K.S. Mathematical Modeling, Case Studies and Projects, 4th Edition, Kluwer Academic Publishers, New Delhi, India. (2009), 233-329
- [5] Rekha V. J. “Optimization Techniques for Transportation Problems of Three Variable”, *IOSR Journal of Mathematics*. Volume 9, Issue 1, (2013), 46-50.
- [6] Shah N.H., Gor R.M., & Soni, H. Operations Research, 4th Edition, PHI Learning Private Limited, New Delhi, India. (2010), 195-240.
- [7] Sudhakar, V.J., Arunnsankar N., & Karpagam T. A new approach for find an Optimal Solution for Transportation Problems, *European Journal of Scientific Research*. (2012), 68, 254-257.