Analytical Solution of the Effect of Suction/Injection on Transient Natural Convection Micro-Gas Flow between two Vertical Parallel Plates: A Time-Periodic Regime

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Abstract: *This paper analyzed the hydrodynamic and thermal behavior of an unsteady fully developed natural convection flow, in a vertical parallel micro-porous-channel (whose boundaries are heated sinosoidally), in the presence of suction/injection, with velocity slip and thermal jump at the walls.The exact solutions of the momentum and energy equations as well as the expressions for skin friction and thermal flux at the walls are obtained. The variation of temperature and velocity with respect to frequency of the driving force, Knudsen number, suction/injection parameter, combined values of frequency of wall's temperature oscillation and time* $(\omega\tau)$*, and that of skin friction and heat flux with respect to suction/injection parameter and* $\omega\tau$ *are discussed.Numerical values of skin friction, heat flux, temperature and velocity are computed. It is found that injection acceleratesand suction retards the flow.*

Keywords: Suction/Injection, sinusoidal temperature, frequency of the driving force, Knudsen number

List of symbols

C_p	Specific heat
k	Thermal conductivity
Kn	Knudsen number $(=\lambda / L)$
L	Reference length
T	Temperature
T_∞	Ambient temperature
T_W	Wall temperature
S	Dimensionless Suction/Injection velocity $(=V_o \varepsilon L / \mu)$
Pr	Prandl number $(=\upsilon / \alpha)$
t	Time
t_r	Reference time $(=L^2 / \upsilon)$
$\omega\tau$	Combined dimensionless frequency of wall's temperature oscillation with dimensionless time
ϖt	Frequency of wall's temperature oscillation with time
U	Dimensionless axial velocity $=(u / u_r)$
u	Axial velocity (in x -direction)
u_r	Reference velocity $(=g\beta L^2 (T_w - T_\infty) / \upsilon)$
V	Complex solution function for velocity
V_o	Suction/injection
W	Complex solution function for temperature
X	Dimensionless axial coordinate $(=x / L)$
x	Axial coordinate
Y	Dimensionless transverse coordinate $(=y / L)$
y	Transverse coordinate

Greek symbols

α	Thermal diffusivity
β	Coefficient of thermal expansion
γ	Specific heat ratio $(\rho C_p / C_\upsilon)$
λ	Mean-free-path length
ρ	Density
μ	Dynamic viscosity
υ	Kinematic viscosity
θ	Dimensionless temperature $(=T - T_\infty / T_W - T_\infty)$
σ_T	Thermal accommodation coefficient $(=0.7)$
ϖ	Frequency
ω	Dimensionless frequency $(=\varpi / \omega_r)$
ω_r	Reference frequency $(=\upsilon / L^2)$
τ	Dimensionless time $(=t / t_r)$

1.0 Introduction:

Natural convection flows are mechanisms or types of transport in which the fluid motion is initiated by density differences, which is caused by difference in temperature gradients. In this process, the fluid surrounding heat source, when heated, becomes less dense, and consequently rises to allow the cooler one replace it. As a result of continuous heating, the hotter fluid continuously rises while the cooler one falls; this forms convection current (i.e. the process of transferring heat energy from the bottom to the top of the convection cell). In short, buoyancy effect (a result of differences in density of the fluid) is the driven force for natural convection.

Natural convection has numerous applications in nature; such as in see-wind formation and oceanic currents. Free air cooling without the aid of fan is a very common industrial application of natural convection. In the field of engineering also, natural convection heat transfer is essential in cooling of high voltage electrical power transformers, heating of houses by electrical base-board heaters, cooling of electronic devices such as chips and transistors by finned heat sinks, cooling of reactor cores in nuclear power plant, and the rest.

It has been found in the literature that, natural convection rate depends upon the physical constants of the fluid density, viscosity, thermal conductivity, specific heat at constant pressure, co-efficient of thermal expansion, diameter or length, temperature difference and gravitational acceleration [1].

Heat transfer in micro-channels with sinusoidal temperature has attracted lot of growing interest across the globe; this is so because of its application in many engineering and industrial processes, and in many natural phenomena. It is also found to have played an important role in the automatic control systems and in electronic and electrical components subjected to periodic heating.

In the year 1966, a uniform heat flux which varies periodically in time with square pulses for right vertical wall was investigated[2]. The case in which the surfaces temperature slightly varies about a mean value, which is higher than the ambient temperature was studied in[3], where they restricted their results to small amplitudes of the surface temperature variation. The same problem was solved using a slightly different perturbation expansion, though their results were also restricted to small amplitude[5]. Laminar natural convection about vertical plates with oscillatory surface temperature was studied in[4], and they overcome the restrictions on the amplitude using the finite difference method.

More recently, mixed convection flow in a vertical tube filled with porous material with periodic boundary conditions were studied in[5], where they found the existence of two local maxima, one near the surface of the tube and the other on the tube axis. The effect of frequency of fluctuating driving force on basic gaseous micro-flows was studied in[6], where they consider four flow cases, namely; Couette flow, Poiseuille flow, Stokes second problem and natural convection. They found the effect of the frequency of the driving force, ω on velocity and temperature to be similar, while increasing the frequency of the driving forcewas found to decrease the local velocity and temperature of the flow. Free convective flow between vertical porous plates with periodic heat input was analyzed in[7], the temperature was found to be higher near the plate with injection, while velocity was found more enhanced near the plate with suction.

In another instance, suction/injection of fluid through bounding surfaces, as in mass transfer cooling can significantly change the flow field, as a consequence, affects the rate of heat transfer from the bounding surfaces. In general, injection tends to decrease the skin-friction and heat transfers co-efficients whereas suction acts in the opposite manner. Injection/suction of fluid through porous heated or cooled surfaces is of general interest in practical problems involving film cooling, control of boundary layers, etc. This can lead to enhanced heating or cooling of the system and can help to delay the transition from laminar to turbulent flow [8]. The role of suction/injection on steady fully developed mixed convection flow in a vertical parallel plate micro-channel was studied in[9], where one of the walls is considered to be cold and the other to be opposite. They drafted the conclusions that with increase in Knudsen number, the fluid temperature on the cold porous plate increases, while it decreases on the hot plate, and upon increasing the Knudsen number, the velocity and the velocity slip on both walls of the micro-channel increases. Natural convection flow in a vertical micro-channel with suction/injection was analyzed in[10]. They reported from their work that, due to a decrease of velocity and temperature within the channel, suction/injection increase the volume flow rate and decrease the heat transfer rate. Same authors considered effect of viscous dissipation on natural convection flow between two vertical parallel plates with time-periodic boundary conditions[11], where they reported that for relatively small Prandtl number, viscous dissipation within the channel increases the temperature of the fluid.

To the best of authors' knowledge, heat transfer in micro-channels with sinusoidal temperature, in the slip flow regime, where the temperature gradient is sinusoidal has never been conducted in the literature, by any researcher. Consequently, the natural convection studied by Haddad et al [6] is extended in this study by considering the flow assuming itself between two permeable boundaries.

2.0 Formulation of the problem

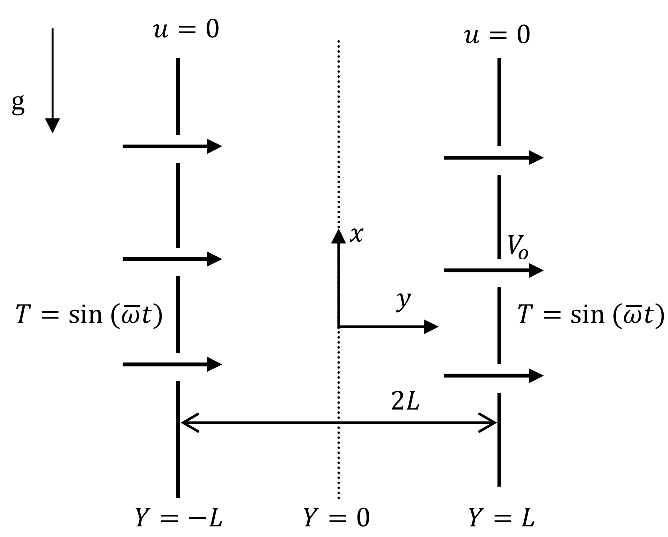

Fig. 1:*Schematic Diagram of the problem*

Consider the transient hydro-dynamically natural convection of a basic micro-gas in the slip flow regime between two parallel, permeable vertical plates of infinite lengths, where the two plates are subjected to sinusoidal temperature variation. Basically, the flow regimes are classified into four different categories: continuum flow $Kn < 10^{-3}$, slip flow regime $10^{-3} \le Kn < 10^{-1}$, transition flow $10^{-1} Kn \le 10^{1}$ and the free molecular flow $Kn > 10^{1}$; these classifications are based on the values of Knudsen number Kn [12, 13]. For the slip flow regime, the values of Kn were taken between 10^{-3} and 10^{-1} as reported in the literature. The tangential momentum accommodation coefficient σ_{υ} determines the degree of slip in the system which depends on the boundary surface conditions and the working fluid, and it has been reported experimentally to be between the ranges of $0.2-0.8$. Its lower limit of applicability applies to extremely slip condition while the upper limit applies to practical and experimental purposes. Value of tangential momentum accommodation is taken as $\sigma_{\upsilon} = 0.7$ since extremely slip condition isn't considered in this case. The flow is assumed to be fully developed hydro-dynamically, and hence, the change in velocity and temperature with respect to x is zero. i.e. $\partial u / \partial x = \partial T / \partial x = 0$.

Based on the facts given above, the governing equations and boundary conditions are modeled as follows:

Governing equations:

$$\rho\left(\frac{\partial u}{\partial t} + V_o \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} + g\rho\beta(T - T_\infty) \tag{2.1}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + V_o \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2} \tag{2.2}$$

Boundary conditions:

$$u(t,-L) = \frac{2-\sigma_v}{\sigma_v} \lambda \left.\frac{\partial u}{\partial y}\right|_{y=-L} \tag{2.3}$$

$$u(t,L) = -\frac{2-\sigma_v}{\sigma_v} \lambda \left.\frac{\partial u}{\partial y}\right|_{y=L} \tag{2.4}$$

$$T(t,-L) - \sin(\varpi t) = -\frac{2-\sigma_T}{\sigma_T} \frac{\lambda}{\Pr} \frac{2\gamma}{1+\gamma} \left.\frac{\partial T}{\partial y}\right|_{y=-L} \tag{2.5}$$

$$T(t,L) - \sin(\varpi t) = -\frac{2-\sigma_T}{\sigma_T} \frac{\lambda}{\Pr} \frac{2\gamma}{1+\gamma} \left.\frac{\partial T}{\partial y}\right|_{y=L} \tag{2.6}$$

Note that the symmetric conditions applied in[6] *are removed, this is due to the fact that they cannot be forced on flow involving suction and injection.*

Considering the following dimensionless variables:

$$\tau = \frac{t}{t_r}, \omega = \frac{\varpi}{\omega_r}, Y = \frac{y}{L}, \; Kn = \frac{\lambda}{L}, \; S = \frac{V_o L}{\mu}, \; \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; \Pr = \frac{\upsilon}{\alpha}, U = \frac{u}{u_r}, \; \alpha = \frac{k}{\rho C_p} \tag{2.7}$$

We substitute Eq. (2.7) into Eq. (2.1-2.6) above, we obtained:

$$\frac{\partial U}{\partial \tau} + S \frac{\partial u}{\partial Y} = \frac{\partial^2 u}{\partial Y^2} + \theta \tag{2.8}$$

$$\frac{\partial\theta}{\partial\tau}+S\frac{\partial\theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial Y^2} \tag{2.9}$$

Subjected to the following boundary conditions:

$$U(\tau,-1)=\frac{2-\sigma_v}{\sigma_v}Kn\frac{\partial U}{\partial Y}\bigg|_{Y=-1} \tag{2.10}$$

$$U(\tau,1)=-\frac{2-\sigma_v}{\sigma_v}Kn\frac{\partial U}{\partial Y}\bigg|_{Y=1} \tag{2.11}$$

$$\theta(\tau,-1)-\sin(\omega\tau)=\frac{2-\sigma_T}{\sigma_T}\frac{Kn}{\Pr}\frac{2\gamma}{1+\gamma}\frac{\partial\theta}{\partial Y}\bigg|_{Y=-1} \tag{2.12}$$

$$\theta(\tau,1)-\sin(\omega\tau)=-\frac{2-\sigma_T}{\sigma_T}\frac{Kn}{\Pr}\frac{2\gamma}{1+\gamma}\frac{\partial\theta}{\partial Y}\bigg|_{Y=1} \tag{2.13}$$

Where $Pr$is the Prandtl number, $Kn\ (=\lambda/L)$ is the Knudsen number and S is the suction/injection velocities. Positive values of $S\ (=V_o L/\mu)$ taken represent injection velocities while the opposite is taken for suction velocity. This type of model has captured lots of interest globally because of its numerous applications, especially in the field of engineering. Some of its applications are essential in the design of thrust bearing and radial diffusers.

3.0 Analytical Analysis

An exact solution for this problem is possible by assuming the following complex solution:

$$U(\tau,Y)=\mathrm{Im}\{\exp(i\omega\tau)V(Y)\}\text{ and }\theta(\tau,Y)=\mathrm{Im}\{\exp(i\omega\tau)W(Y)\} \tag{3.1}$$

Where $'\mathrm{Im}'$represents the imaginary part of the complex solution and$i=\sqrt{-1}$. Differentiating Eq. (3.1) and substituting them into the governing equations and boundary conditions, we transformed them into ordinary differential equations whose solutions are:

$$V(Y)=\frac{1}{b[\cosh(\delta_7)+a\delta_7\sinh(\delta_7)]}\left(\frac{\cosh(Y\delta_9)[c\delta_7\sinh(\delta_7)+\cosh(\delta_7)]}{\cosh(\delta_9)+c\delta_9\sinh(\delta_9)}-\cosh(Y\delta_7)\right) \tag{3.2}$$

$$W(Y)=\frac{\cosh(Y\delta_7)}{\cosh(\delta_7)+a\delta_7\sinh(\delta_7)} \tag{3.3}$$

Substituting Eq. (3.2-3.3) into Eq. (3.1), we obtained the following solutions:

$$U(\tau,Y)=im\left\{\frac{\exp(i\omega\tau)}{b[\cosh(\delta_7)+a\delta_7\sinh(\delta_7)]}\left(\frac{\cosh(Y\delta_9)[c\delta_7\sinh(\delta_7)+\cosh(\delta_7)]}{\cosh(\delta_9)+c\delta_9\sinh(\delta_9)}-\cosh(Y\delta_7)\right)\right\} \tag{3.4}$$

$$\theta(\tau,Y)=\mathrm{Im}\left\{e^{i\omega\tau}\left(\frac{\cosh(Y\delta_7)}{\cosh(\delta_7)+a\delta_7\sinh(\delta_7)}\right)\right\} \tag{3.5}$$

Where $a=\frac{2-\sigma_T}{\sigma_T}\frac{Kn}{\Pr}\frac{2\gamma}{1+\gamma},\ b=\delta_7^2-S\delta_7-i\omega,\ c=-\frac{2-\sigma_v}{\sigma_v}Kn$ (3.6)

$$\delta_7=\frac{S\Pr+\sqrt{(S\Pr)^2+4i\omega\Pr}}{2},\ \delta_8=\frac{S\Pr-\sqrt{(S\Pr)^2+4i\omega\Pr}}{2}\ and\ \delta_9=\frac{S+\sqrt{S^2+4i\omega}}{2} \tag{3.7}$$

From Eq. (3.6) and (3.7), the skin friction and heat flux on the boundaries were obtained respectively as follows:

$$\tau_{-1} = \left.\frac{\partial U}{\partial Y}\right|_{Y=1} = im\left\{\frac{-\exp(i\omega\tau)}{b\left[\cosh(\delta_7) + a\delta_7 \sinh(\delta_7)\right]}\left(\frac{\delta_9 \sinh(\delta_9)\left[c\delta_7 \sinh(\delta_7) + \cosh(\delta_7)\right]}{\cosh(\delta_9) + c\delta_9 \sinh(\delta_9)} - \delta_7 \sinh(\delta_7)\right)\right\} \quad (3.8)$$

$$\tau_{1} = \left.\frac{\partial U}{\partial Y}\right|_{Y=1} = im\left\{\frac{\exp(i\omega\tau)}{b\left[\cosh(\delta_7) + a\delta_7 \sinh(\delta_7)\right]}\left(\frac{\delta_9 \sinh(\delta_9)\left[c\delta_7 \sinh(\delta_7) + \cosh(\delta_7)\right]}{\cosh(\delta_9) + c\delta_9 \sinh(\delta_9)} - \delta_7 \sinh(\delta_7)\right)\right\} \quad (3.9)$$

$$q_{-1} = \left.\frac{\partial \theta}{\partial Y}\right|_{Y=1} = \mathrm{Im}\left\{e^{i\omega\tau}\left(\frac{-\delta_7 \sinh(\delta_7)}{\cosh(\delta_7) + a\delta_7 \sinh(\delta_7)}\right)\right\} \quad (3.10)$$

$$q_{1} = \left.\frac{\partial \theta}{\partial Y}\right|_{Y=1} = \mathrm{Im}\left\{e^{i\omega\tau}\left(\frac{\delta_7 \sinh(\delta_7)}{\cosh(\delta_7) + a\delta_7 \sinh(\delta_7)}\right)\right\} \quad (3.11)$$

4.0 Discussion and Results

In order to have physical insight of the problem, MATLAB programming is employed to generate and analyze results. The problem is found to be absolutely symmetric despite the fact that it is not forced to be so, this is due to same boundary conditions applied on both walls. On the graphs, for clear visibility, we consider the center of the channel (i.e. *y=0*) to one of the walls (at *y=1*),with a view to reflect what happens on the other wall *(at y=-1)*. For the sake of accuracy, our results were compared with those in[6] and they were found to be in excellent agreement. When values of suction/injection are assumed to be zero, our results reverse to those in[6]. This validates the accuracy of the results.

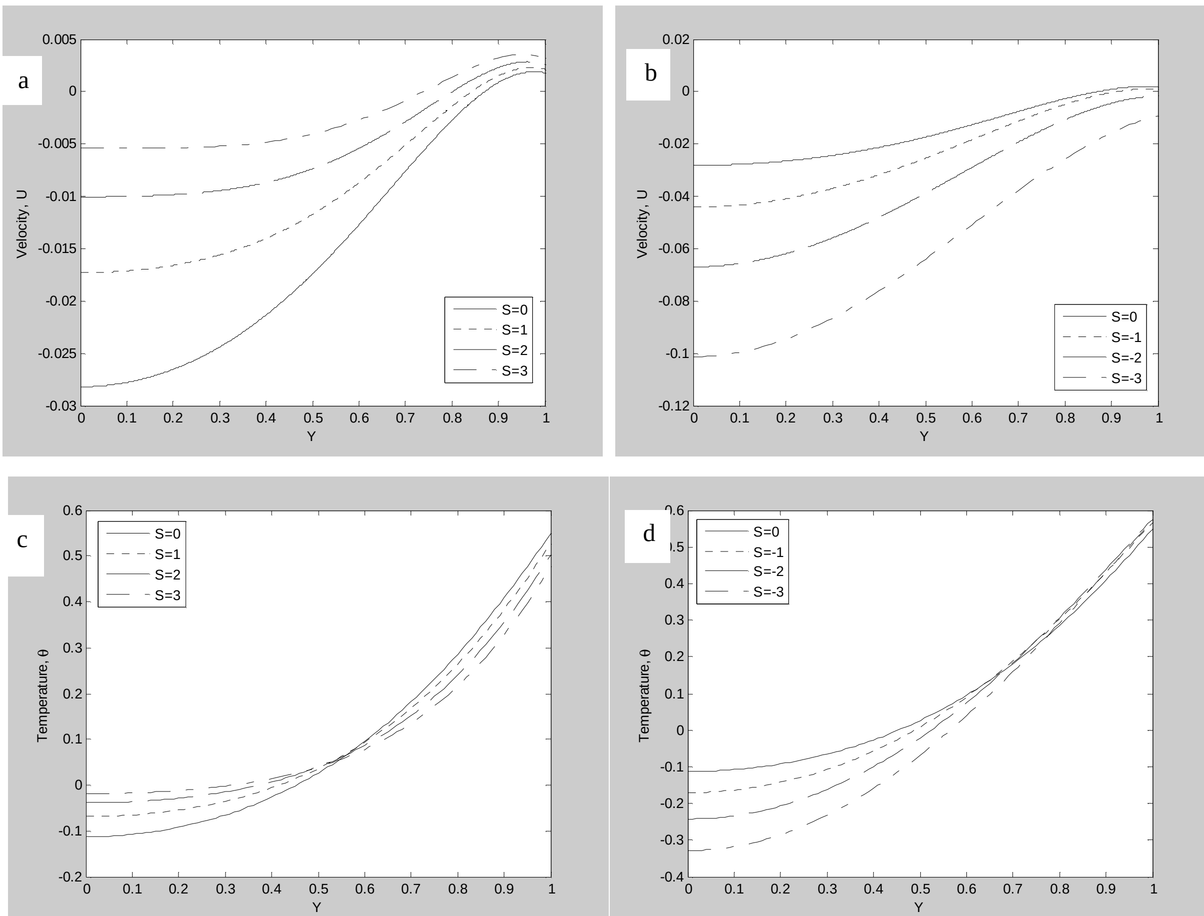

Fig. 2: *Velocity and Temperature Profiles for* $\Pr = 0.71,\ \omega\tau = \pi/2,\ Kn = 10^{-1},\ \gamma = 1.4,\ \omega = 10.$

Fig. 2 (a) highlights the effect of injection on the fluid velocity curve from the wall, to the center of the channel. It is deduced from the figure that increasing injection velocity increases the fluid flow. The velocity is minimal at $y = 0$ and increases up to a particular length of the channel ($y = 0.85$ to 0.95), then decreases to the wall.

Fig. 2 (b) illustrates the effect of suction on the velocity curve as y increases from center, through the walls of the channel. On the walls, velocity is found to be very high when compared to the center of the channel. An increase in suction velocity generally causes an increase in fluid velocity.

Fig. 2 (c) illustratively interprets the effect of injection on the temperature curve as y decreases from the wall through the channel's center. Temperature is found to be very high on the wall when compared to the center of the channel; this is because of the sinusoidal temperature applied on the walls. An increase in injection velocity causes a decrease in temperature at the wall ($at\ y = 1$) through $y = 0.55$ approximately. At this point ($y = 0.55$), there is a switch, and increase in injection velocity causes an increase in temperature up to the center of the channel. Temperature varies proportionally with the length of the channel (using *y=0* as a reference point).

Fig. 2 (d) illustrates suction effect on the temperature distribution of the flow. From the center, temperature also varies directly with the length of the channel. This is to say, for every value of suction velocity used, it is observed that the temperature increases with the length of the channel and vice versa. It has been noticed that an increase in suction velocity leads to a decrease in temperature up to a particular length in the channel ($y = 0.65$ and $y = 0.75$) where a temperature switch is observed. At this point, an increase in suction velocity increases the temperature of the system.

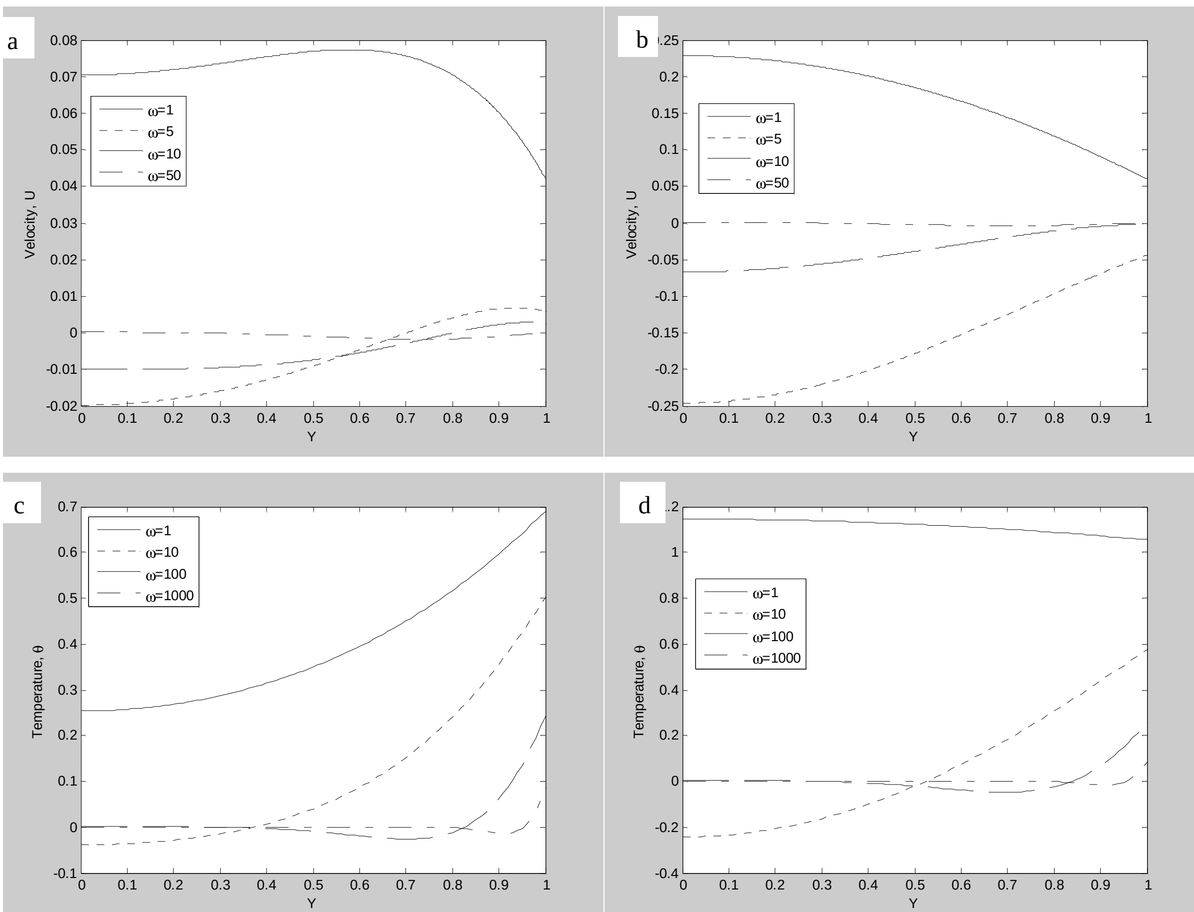

Fig. 3: *Velocity and Temperature Profiles for* $\mathrm{Pr} = 0.71,\ \omega\tau = \pi/2,\ Kn = 10^{-1},\ \gamma = 1.4,\ (a)\,S = 2\,(b)\,S = -2$
$(c)\,S = 2\ \ (d)\,S = -2$

Fig. 3 (a) represents the velocity profile resulting from different values of the frequency of the driving force,$\omega$when injection velocity and other parameters are kept constants. For$\omega = 1$ and$\omega = 5$ respectively, the velocity is found to be maximum and minimum. As ω increases from 5, the velocity also increases at *y=0* to about *y=0.5 to y=0.8*. About this point, there is a switch, and increase in ω decreases the flow velocity at *y=1*.

Fig. 3 (b) depicts the effect of angular velocity on velocity of the flow for constant value of suction velocity. For $\omega = 1$, the velocity is higher, though it attains its maximum at *y=0*. The velocity also found to be lowest when $\omega = 5$, and then increases with increase in ω.

Fig. 3(c) indicates the effect of angular velocity on the sinusoidal temperature of the flow. It is found that temperature decreases with increase in angular velocity. Highest temperature is observed on the wall which decreases through the center of the channel. It is also comprehended from the figure that, as temperature increases with increase in angular velocity, there is a crossover at $y = 0.35$ for the values of $\omega = 10, 100$ and 1000, and at $y = 0.83$ for the values of $\omega = 100$ and 1000. The effect of angular velocity on the temperature is more pronounced for small values of ω than for the higher values.

Fig. 3 (d) displays the effect of angular velocity on the temperature of the system when suction velocity and other parameters are kept constants. Temperature is found to decrease with increase in angular velocity. For the value $\omega = 1$, it is found that the temperature is higher at the center and decreases to the wall. While for $\omega = 10, 100$ and 1000, the temperature is found to be higher on the wall and decreases sideward to the center of the channel. As the temperature decreases sideward from the wall, a crossover temperature is observed at about $y = 0.35$ and 0.83 for values of $\omega = 100$ and 1000, at about $y = 0.48$ for $\omega = 10$ and 100, and at about $y = 0.53$ for $\omega = 10$ and 1000.

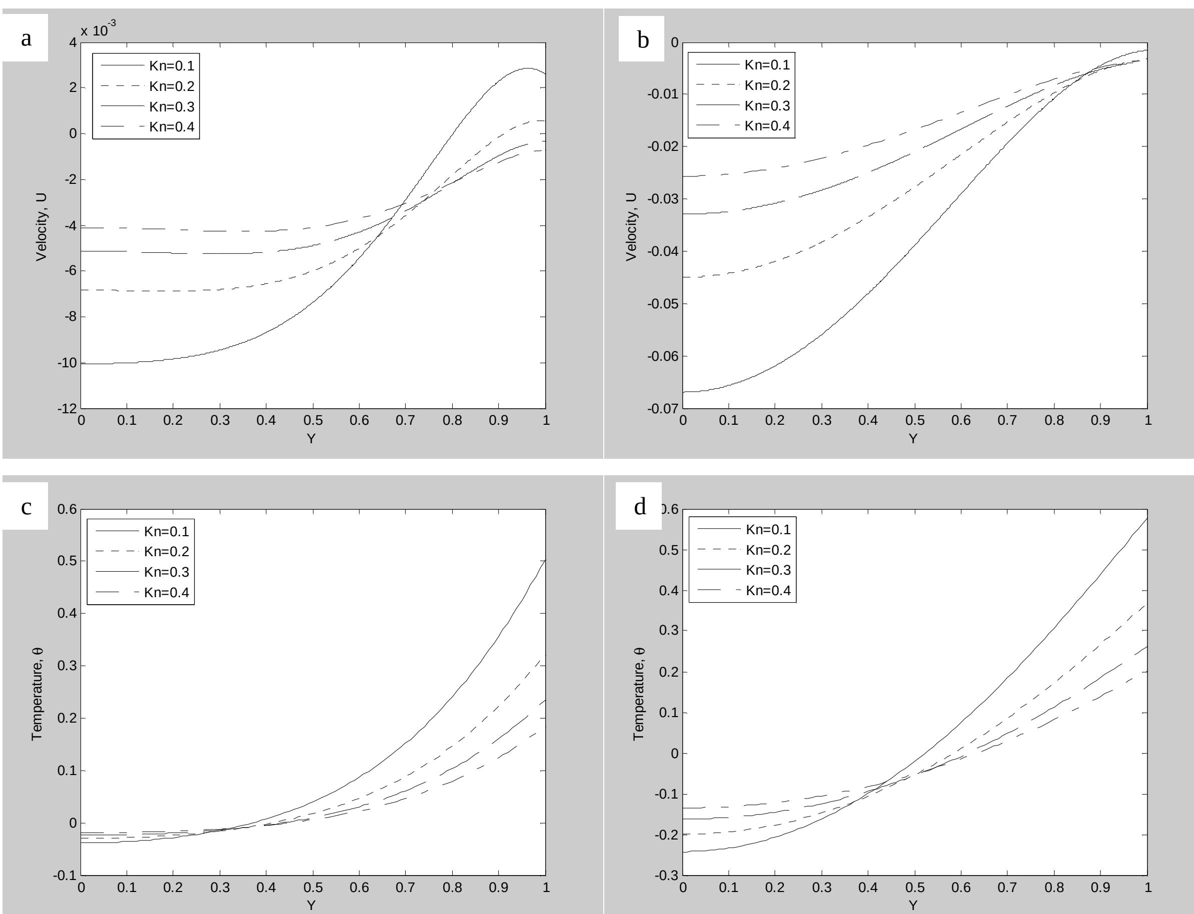

Fig 4: *Velocity and Temperature Profiles for* $\Pr = 0.71,\ \omega = 10,\ \omega\tau = \pi/2,\ \gamma = 1.4,$
$(a)\ S = 2\ \ (b)\ S = -2\ \ (c)\ S = 2\ \ (d)\ S = -2$

Fig. 4 (a)captures the effect of Knudsen number on the velocity curve as y increases from $y = 0$ to $y = 1$. Increase in Knudsen number retards the flow at the center up to a particular point (at about $y = 0.6 - 0.75$)where there is a switch of velocity which reverses on the wall, the velocity of the flow.

Fig. 4 (b) displays the effect of varying Knudsen number on the velocity of the fluid. The velocity increases with the length of the channel. Higher velocity is recorded at $y = 1$ for fluid with$Kn = 0.1$, and decreases with increase inKn. As velocity decreases downward from $y = 1$, there is a switch at about$y = 0.95 - 0.85$. This reverses the velocity at $y = 0$, the center of the channel; that is to say, increasing Kn leads to increase in velocity at$y = 0$.

Fig. 4 (c) captures the effect of increase in Knudsen number on the temperature distribution in the flow for constant injection velocity. It is found on the wall that the temperature is higher for the fluid with Knudsen number 0.1 and decreases with increase in Knudsen number. The temperature is also higher on the slip wall and decreases proportionally sideward and approaches zero at about$y = 0.28$.

Fig. 4 (d) illustrates the effect of increase in Knudsen number on the temperature distribution in the flow for constant suction velocity. It is obviously observed on the figure that temperature is higher at $y = 1$ and decreases sideward to approach zero at about$y = 0.6$. It is also found that fluid with small Knudsen number freezes more and faster than that with high Knudsen number.

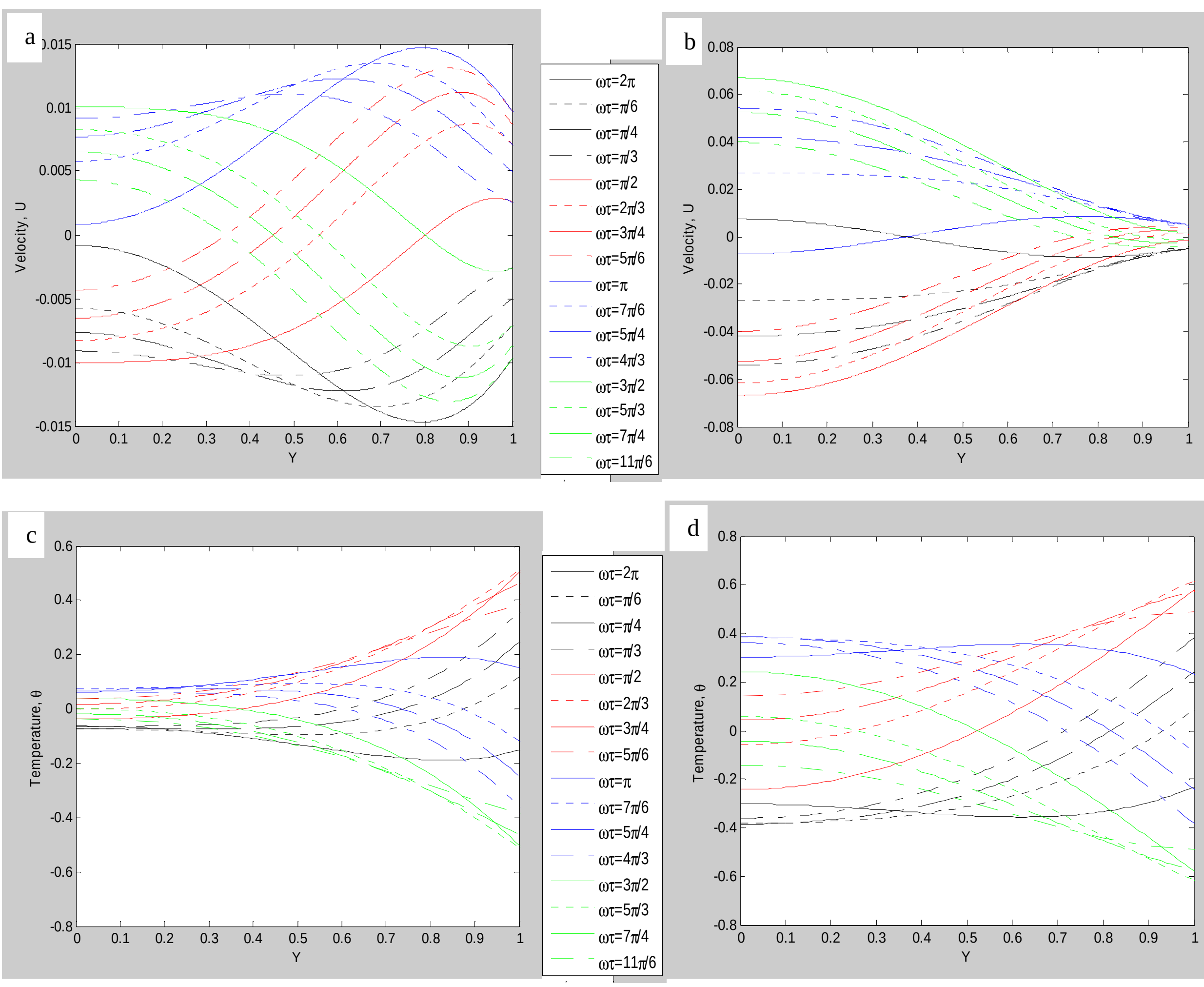

Fig 5: *Velocity and Temperature Profiles for* $\Pr = 0.71,\ \omega = 10,\ Kn = 10^{-1},\ \gamma = 1.4,$

$(a)\ S=2\ (b)\ S=-2\ (c)\ S=2\ (d)\ S=-2$

It's of immense importance to discuss about the optimal and minimal velocities of the flow with respect to the values of $\omega\tau$, (i.e. value of $\omega\tau$ for which the velocity is optimal and that for which the velocity is minimal). For the sake of analysis, special values of $\omega\tau (i.e\ 2\pi, \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3, 3\pi/4, 5\pi/6, \pi, 7\pi/6,\ 5\pi/4, 4\pi/3, 3\pi/2, 5\pi/3, 7\pi/4\ and\ 11\pi/6)$were considered. The velocity curve is found to be optimal/minimal with difference of π between the respective values of $\omega\tau$, so also oscillate between the optimal and minimal velocities as the values of $\omega\tau$ increased periodically. This is the same for the temperature of the flow. It is also important to note that, values whose differences areπ were generally found to be symmetrical about the velocity/temperature axes.

Fig. 5 (a) showshow values of $\omega\tau$affect the velocity of the flow for constant injection velocity. It's found on the figure that, the fluid attains its maximum and minimum velocities at the centerof the channel when $\omega\tau = 3\pi/2$ and $\pi/2$ respectively, while on the wall,this is attainedwhen $\omega\tau = \pi$ and2π.

Fig. 5 (b) visualizes how velocity changes with values of $\omega\tau$for constant suction velocity. It is clearly observed at the center of the channel that, the fluid movesrespectively with maximum and minimum velocities when$\omega\tau = 3\pi/2$ and $\pi/2$,while on the wall, it moves with such respectivevelocities for the values of$\omega\tau = 7\pi/6$and$\pi/6$.

Fig. 5 (c) indicates how temperature in the flow changes with respect to values of $\omega\tau$ for constant injection velocity. The temperature of the flow is found to be maximum on the channel's wall when$\omega\tau = 7\pi/6$ and minimum when$\omega\tau = \pi/6$, while at the center of the channel, the temperature is found to be maximum and minimum when$\omega\tau = 2\pi/3$ and $5\pi/3$ respectively.

Fig. 5 (d) figures out the effect of $\omega\tau$ on the temperature distribution of the flow for constant suction velocity. Maximum temperature on the wall and at the center are achieved respectively when$\omega\tau = 2\pi/3$ and $5\pi/4$, while on the wall and at the centre, the optimum values are found to be when$\omega\tau = 5\pi/3$ and $\pi/4$respectively.

These optimum temperatures found are more pronounced for suction when compared with injection.

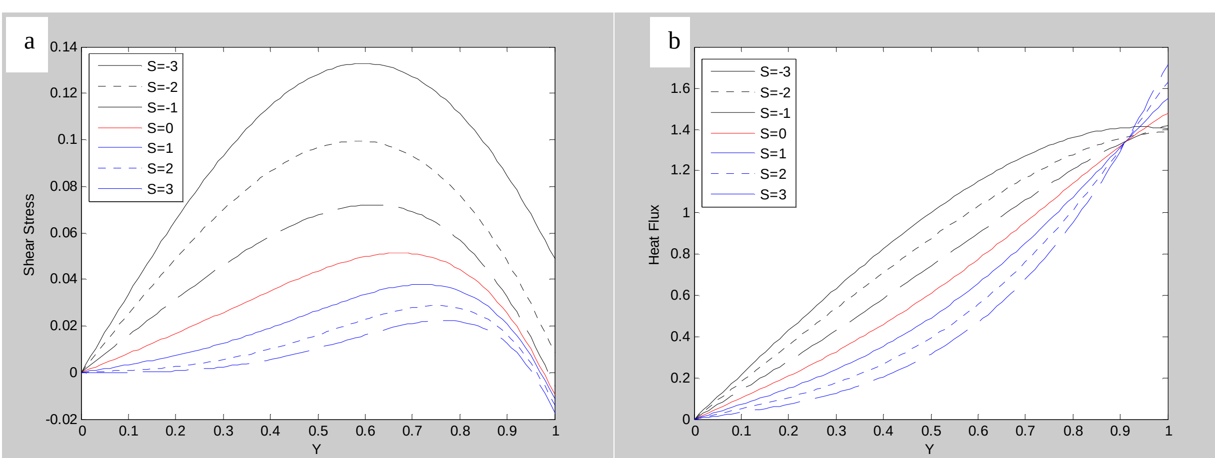

Fig 6: *Shear Stress and Heat Flux Profiles for* $\Pr = 0.71,\ Kn = 10^{-1},\ \gamma = 1.4,\ \omega = 10,\ \omega\tau = \pi/2$

Fig. 6 (a) and (b) above respectively highlighted the effect of suction/injection on shear stress and heat fluxof the flow as y advances to the wall, from the center of the channel. Shear stress is found to have increased with suction and decreased with injection, and increasing suction velocity increases the shear stress whileit decreases with increasing injection velocity. The shear stress alsoincreases up to a particular point in the channel (at about $y = 0.4$ to $y = 0.8$) where it is found to be maximum, then decreases to the wall.It is also found that suction increases the heat flux up to a particular point in the channel (i.e. at about $y = 0.96$ to 0.98) where there is a switch, this reverses the heat flux to the wall of the channel. Injection decreases the heat flux, and increasing injection velocity generally lead to a decrease in heat flux of the flow. Heat flux is found to decrease with the length of the channel, up to a particular point (i.e. at about $y = 0.93$) where there is a switch. From this point, increasing injection velocity increases the rate of heat flux to the wall of the channel.

Table 1: *Numerical values of skin friction, heat flux, velocity and Temperature*

$(\Pr = 0.71,\ \omega = 10,\ Kn = 10^{-1},\ \gamma = 1.4,\ \omega\tau = \pi/2)$

S	Y	τ_Y	q_Y	V	θ		S	Y	τ_Y	q_Y	V	θ
0	1.0	-0.4909	0.2615	0.0018	0.5490		0	1.0	-0.4909	0.2615	0.0018	0.5490
1	1.0	-0.2412	0.6849	0.0021	0.5266		-1	1.0	-0.8547	-0.1443	0.0011	0.5671
2	1.0	-0.2271	1.0153	0.0026	0.5022		-2	1.0	-0.3228	-0.1803	-0.0016	0.5765
3	1.0	-0.1610	1.2979	0.0032	0.4770		-3	1.0	-0.0833	-0.1059	-0.0091	0.5719

5.0 Conclusion

The following conclusion were drawn from this study:

1. Suction and injection act inopposite manner on velocity and temperature of the flow. Suction is found to decrease the velocity of the flow, while injection does the reverse. On the temperature of the flow, though suction/injection act in opposite manners, their effects at the center and on the wall are also opposite; this is due to a temperature switch at particular point of the channel. Suction decreases the temperature at the center of the channel, while on the wall, this effect is reversed. Opposite of this is found for injection.
2. For both suction/injection, the velocity and temperature of the flow increase for value of frequency of the driving force, $\omega = 5$ and decrease for values higher than that. This effect, on velocity of the flow, it's found to be more enhanced at the center of the channel, while for temperature of the flow, it's more enhanced on the wall.
3. Increase in Knudsen number generally results to a decrease in velocity and temperature of the moving fluid at the center of the channel and the reverse on the wall, even though the velocity is higher with injection, while the temperature is more enhanced with suction.
4. The velocity and temperature curves are found to be optimal/minimal with difference of π between the respective values of $\omega\tau$, so also oscillate between these optimal and minimal velocities as the values of $\omega\tau$ increased periodically. These resonance frequencies are found not to be unique, but depend upon the length of the channel.It is also found that, values whose differences are π were symmetrical about the velocity/temperature axes.

5. Shear stress increases with suction and decreases with injection. Suction increases heat flux up to a particular point in the channel where the heat flux switches. From this point, suction retards the flow to the wall of the channel, while injection exactly does the reverse.

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