Linearization of a Model Equation for Structural Vibration Problems using Differential Forms

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Abstract

In this paper, the linearizing point transformation for themodel equation of structural vibration problems using the method of differential forms is obtained.

Key Words: *Point transformation, Differential forms, Linearization, Model equation, Structural Vibration Problems, Second order ordinary differential equations.*

1. Introduction

The problem of linearization of ordinary differential equations has a long history. It attracted attention of mathematicians such as Sophus Lie and E. Cartan [1]. The first linearization problem for ordinary differential equations was solved by Sophus Lie in 1883 [2].

The linearization problem for a second order ordinary differential equation was also investigated with respect to differential forms in[3]. It is important to state that, linearization method in general, has to do with point transformation (PT) and non-point transformation (NPT) [4]. Point transformation preserves the integrability of the equation and its Lie symmetry structure [5], and hence the reason for the use of point transformation.

The model equation for structural vibration problemswas also treated in [6] using themethodof classification of second order ordinary differential equations admitting Lie groups of fibre-preserving symmetries.

In this paper, the linearizing point transformation for the model equation of structural vibration problems using the method of differential forms given in [7], is constructed with its solution.

2. Construction of the Point Transformation

The method is explicitly stated in [7] and following the procedure in [7], we can construct the point transformation of the model equation for structural vibration problems.

The equation is given as

$$y'' = (x-1)y'^3$$

which can be written as

$$y'' + (1-x)y'^3 = 0 \qquad (1)$$

with the coefficients

$$f_0 = f_1 = f_2 = 0, f_3 = 1-x$$

satisfying the linearizability conditionsstated in [7], hence,the equation (1) is linearizable. We can therefore proceed to construct the 3×3 matrix M to have

$$M = \begin{bmatrix} 0 & 0 & 0 \\ (1-x)dy & 0 & -dy \\ -dx & dy & 0 \end{bmatrix}.$$

Setting $r = \begin{bmatrix} U \\ V \\ W \end{bmatrix}$,we have

$$dr = \begin{bmatrix} 0 \\ U(1-x)dy - Wdy \\ -Udx + Vdy \end{bmatrix}.$$

Taking $U = 0$we have $dV = -Wdy$and $dW = Vdy$ which are satisfied by

$V = -\sin y$ and$W = \cos y$.

We can now have that $K = \frac{U}{W} = 0$, $L = \frac{V}{W} = -\tan y$, from [7].

We can now construct the matrix Z as

$$Z = \begin{bmatrix} tan\, y\, dy & 0 \\ tan\, y\, dx + (x-1)dy & 2\, tan\, y\, dy \end{bmatrix}.$$

Setting $R = \begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$, then

$$dR = \begin{bmatrix} b\, tan\, y\, dy \\ b\, tan\, y\, dx + [b(x-1) + 2c\, tan\, y]dy \end{bmatrix}$$

so that $db = b\, tan\, y\, dy$.On integration, we have

$$b = k\, sec\, y \tag{2}$$

where k is a constant.

We also note that

$$dc = b\, tan\, y\, dx + [b(x-1) + 2c\, tan\, y]dy. \tag{3}$$

Substituting the value of b into equation (3) we have

$$dc = k\, sec\, y\, tan\, y\, dx + [k\, sec\, y\, (x-1) + 2c\, tan\, y]dy$$

so that

$$c_y = k\, sec\, y\, (x-1) + 2c\, tan\, y \tag{4}$$

and

$$c_x = k\, sec\, y\, tan\, y.$$

Differentiating equation (2) with respect to y we have; $b_y = k\, sec\, y\, tan\, y$, that is $b_y = c_x$.

Integrating $c_x = k\, sec\, y\, tan\, y$ with respect to x, we have

$$c = kx \sec y \tan y + g(y). \tag{5}$$

Differentiating equation (5) with respect to y, we obtain:

$$c_y = kx \sec y + 2kxtan^2 y \sec y + g^{'}(y). \qquad (6)$$

Equating equations (4) and (6), and simplifying we have

$$g^{'}(y) - 2\tan y\, g(y) = -k \sec y. \qquad (7)$$

Using the integrating factor, we can solve equation (7) as follows;

$$P = -2\tan y, Q = -k \sec y$$

so that

$$I.F = e^{-2\int tanydy} = e^{-2lnsecy} = e^{ln\frac{1}{sec^2 y}} = cos^2 y.$$

Therefore $g \times I.F = \int (Q \times I.F)dy + m$ becomes

$$gcos^2 y = \int (-ksecy)cos^2 ydy + m$$

where m is a constant.

Integrating the above and simplifying, we have

$$g = -k \sec y \tan y + msec^2 y. \qquad (8)$$

Substituting equation (8) into equation (5) we obtain

$$c = kx \sec y \tan y - k \sec y \tan y + msec^2 y. \qquad (9)$$

Now considering $F_x = k \sec y$, on integration we have;

$$F = kx \sec y + h(y). \qquad (10)$$

Differentiating equation (10) with respect to y we have,

$$F_y = kx \tan y \sec y + h^{'}(y). \qquad (11)$$

Equating equations (9) and (11) and simplifying we have;

$$h^{'}(y) = -k \sec y \tan y + msec^2 y. \qquad (12)$$

Integrating equation (12), we have

$$h(y) = -k \sec y + m \tan y. \tag{13}$$

Substituting equation (13) into equation (10) and simplifying, we have

$$F = kx \sec y - k \sec y + m \tan y$$

or

$$F = k(x - 1) \sec y + m \tan y.$$

Therefore,

$$X = \tan y, Y = (x - 1) \sec y$$

is the linearizing point transformation of equation (1).

3. Conclusion

In conclusion, the results

$$X = \tan y, Y = (x - 1) \sec y$$

was also obtained in [6] using the methodof classification of second order ordinary differential equations admitting Lie groups of fibre-preserving symmetries. Using differential forms, the same results obtained in [6] is obtained.

The solution thus becomes

$$(\mathrm{x} - 1)\sec y = a \tan y + b$$

where a and b are constants. This can be simply written as

$$x = a \sin y + b \cos y + 1.$$

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