

Mathematical Modeling: A Pivotal Tool For Sustaining National Development

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Abstract

A mathematical model is a representation of a system or scenario that is used to gain qualitative and quantitative understanding of some real-world problems and to predict future behavior. Many developmental challenges that are currently facing this country (Nigeria) could be solved if it is possible to get relevant and adequate mathematical models that could describe them. Some developed countries like United States of America, Japan, Germany and Russia invested heavily on the learning and teaching of Mathematics at all levels of education before reaching their status as world leaders today. The resounding presence of mathematics is evident in all fields of human endeavors. This paper examines the pivotal roles of mathematical modeling and what it plays in all aspect of life, restating it as a central tool for sustainable national development. An illustration of a mathematical corruption model is introduced to show the relevance of mathematical modelling in sustaining national development. It was found that as the corruption increases in the country, the national development decreases using MATLAB software package. Therefore, mathematical modeling is one of the tools needed to transform the Nigerian Economy from a developing nation to a developed nation.

Keywords: Mathematical Modelling, Sustainable Developments, Corruption model, Developing Nations and MATLAB.

1. Introduction

Mathematics is a branch of knowledge that deals with measurements, numbers and quantities. Mathematics is a tool, its knowledge and skills are the bedrock of all societal transformation and transfer of ideas into reality [1]. Each of the diverse branches of mathematics has useful applications on which fields of human endeavors hang. This pivotal position that mathematics occupies makes it a tool for sustaining national development. Mathematical modeling as one of the diverse branches of mathematics has so many roles to play in the sustainability of national development in the country.

A mathematical model is a representation of the real world, characterized by the use of mathematics to represent the parts of the real world that are of interest and the relationships between those parts [2]. Mathematical modeling rests within a Bayesian framework involving the synthesis of all relevant, often disparate, information in the development, implementation and interpretation of the model and its results [3].

Mathematical modeling is thus a veritable tool in solving diverse problems of national importance. It is applicable for instance in economy, security, decision making processes, medicine, population control, etc. Nigeria as a developing country is faced with major challenges in these areas. In order to achieve its developmental goals of ending poverty, promoting sustained, inclusive and sustainable economic growth, making cities and human settlements inclusive, safe, resilient and sustainable among others, mathematical modeling will play a major role [4].

National Development is the ability of a country to improve the social welfare of the people. It involves development of infrastructures such as roads, hospitals, airports, dams, schools, education, health, sports, roads, economy as well as development of its citizenry.

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National development encompasses economic development which is the increase in the standards of living of a Nation's population with sustained growth from a simple, low-income economy to a modern, high-income. So, economic development and growth implies a national development [5]. This paper focuses on the position of mathematical modeling as a pivotal tool for sustainable National Development using Mathematical Corruption Model as an example.

Mathematical Modeling

Oftentimes there is a need to represent a system through a model which will describe all its features. This may be necessary for simplicity and the need for a better description of the system. When a system is such described using tools of mathematics, we talk of a mathematical model. The process of modeling a system by mathematical means is called mathematical modeling.

Types of Mathematical Models

Mathematical models have broad applications and there are many types [6]. Broadly speaking however, we will group these models under the following;

- (i) Descriptive models
- (ii) Optimization models
- (iii) Deterministic and Probabilistic models
- (iv) Specific and General models
- (v) General models

Descriptive models: These models describe phenomenon in a mathematical sense. Natural occurrences and events are modeled mathematically and the relationship between the factors leading to them. They help to provide general better understanding. Descriptive modeling is mainly used by organizations to understand their customers better and for industry-based segmentation.

Optimization models: These types of models are used to make better decisions. It involves a systematic and quantitative process of factoring the many parts of a real situation in order to make the most of the prevailing circumstances. For example, a company may need to decide whether to employ new staff or to lay off employees. Other decision making processes include the need to allocate available resources, flights scheduling in airports, whether to make an investment or not, solving production problems, efficient portfolio optimization and management, budgeting and planning, asset/liability management and so on. The approach to optimization modeling can be summarized in the following steps:

- (i) Describe the problem and gather data
- (ii) Formulate a mathematical model to represent the problem
- (iii) Develop a computer based procedure for deriving solutions to the modeled problem
- (iv) Test and refine the procedure/model
- (v) Implement

Deterministic and Probabilistic models: Deterministic models eliminate all variables of uncertainty. Although many real life situations are probabilistic in nature and require a probabilistic approach to modeling, still we also have a lot of scenarios where outcomes are precisely determined through known relationships among states and events, without any room for random variation. In such models, a given input will always produce the same output, such as in a known chemical reaction or predicting the amount of money in a bank account in a year when the initial deposits and interest rate are known. Other examples of deterministic models are timetables, pricing structures, maps, linear programming models, economic order quantity models, etc.

As a way of thinking, let us say the Super eagles of Nigeria won the African Nations cup. There are several ways we can draw meaning from this. If we are of a deterministic frame of mind, we might say that they won because they are the best National team in Africa. We have assigned cause and effect to the outcome. On the other hand, if we were of the probabilistic view, we would rather ascribe a probability or percentage chance of winning based on a couple of factored considerations.

Specific and General models: Models which are designed specifically for particular situations are called specific models. Most mathematical models are specific in nature as rarely does it happen where two or more exclusive real-world phenomena have the same variables. However, when we do have a model of certainty which can work for general situations we say that such a model is a general one. We often encounter these types of models in deterministic situations.

Formulation of Mathematical Modeling

The development of a mathematical model in general can be considered along the following groups of activities:

- (i) Describing the decision problem
- (ii) Conceptualizing the problem modeling
- (iii) Formulating mathematical model of the problem
- (iv) Solving the modeled problem
- (v) Interpreting the solution

- (vi) Validating the model
- (vii) Engaging with the decision

This description of the model development process is largely generalizable across any modeling application. The flowchart is shown below.

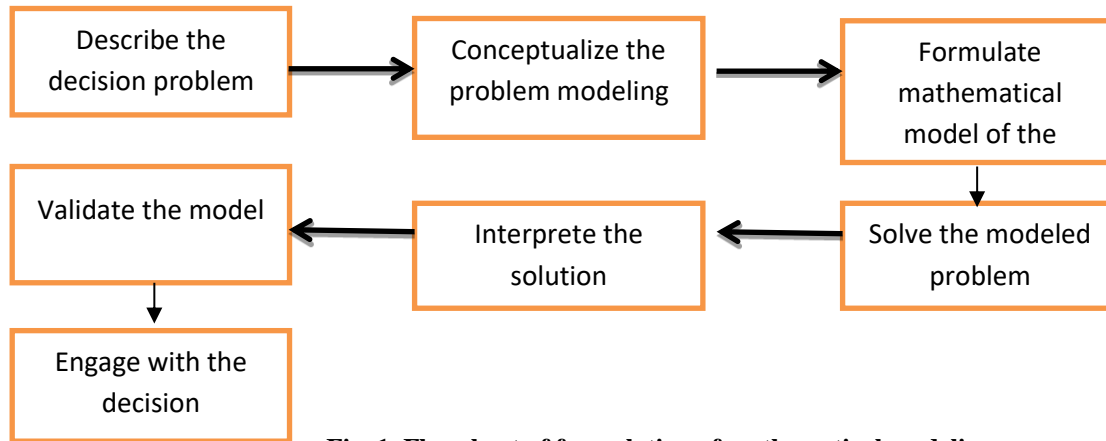


Fig. 1. Flowchart of formulation of mathematical modeling

This is known as the seven stages of mathematical modeling processes.

Roles of Mathematical Modeling in National Development

Many are the roles of mathematical modeling in national development. A few of them are enumerated below.

- (i) Mathematical models are generally easy to use and less expensive than dealing with the actual situation.
- (ii) Mathematical models are consistent tool for evaluation of real life situations.
- (iii) Many financial and economic problems could be modeled mathematically to improve decision making.
- (iv) Mathematical modeling helps in production management for manufacturing industries
- (v) Descriptive modeling for example is used to understand customer behavior and sales projection.
- (vi) Mathematics is used in healthcare modeling to limit the spread of diseases, make better decisions on blood transfusion, treatment, etc.
- (vii) Combat modeling makes use of mathematical models to predict the outcomes of combat and war.
- (viii) Environmental phenomenon like the weather can be modeled mathematically and forecasted properly.
- (ix) Mathematical models are used in ensuring computer and information security [7].
- (x) Mathematical models are effectively employed to solve security problems. For instance, military mathematical models are used in radar, missile defense, search, terrorism, operational reliability, prediction, etc.
- (xi) Mathematical modeling is applied in epidemiology, tumor and cardiac modeling, DNA sequencing and gene technology.
- (xii) It is also useful in the manufacture of medical devices and diagnostics, opto-electronics and sensor technology.
- (xiii) Mathematical models are applied in the area of agriculture to increase food production.
- (xiv) Investors and portfolio managers implement strategies to reduce risk by making use of mathematical modeling.

Examples of Mathematical Modeling

Many examples of mathematical modeling abound and may take different forms. Some of which are; modeling with difference equations, modeling with ordinary differential equations, modeling with partial differential equations and modeling with simulation. Mathematical models do occur in finance, cyber security, environments, corruption, bio-mathematics, agriculture and so on [8]. To show the relevance of mathematical modelling, we shall formulate a mathematical model that shows the relationship between Corruption (C) and National Development (ND) of any field in the country Nigeria. This model shows that as corruption increases, the national development decreases. This is shown in the next sections.

Mathematical Corruption Model

Corruption is a psychological disease that impair integrity, virtue or moral principle. It affects every facet of our lives in this country and which has even disturbed the growth of our national development in this country. In addition, it is an act of inducement by improper means (bribery) to violate duty. The bribe can be in form of money or gifts or other facilities from one person to another. Corruption involves three stages which include primary stage (do the work and gets benefits or more), medium stage (not do the work without benefits) and the last stage (without work but gets benefit or more) [9]. Note that benefits belong to money or gifts or other facilities.

The Mathematical E-virus constant K is defined as the sum of the negative characters of person leading to increase in the corruption model. This constant K is given in the form

$$K = \sum_{i=1}^{13} C_i \text{ for } -1 < K < 1 \tag{1}$$

where

C_1 = Mismanagement of Government funds, C_2 = Insincerity, C_3 = Less Principle, C_4 = Unfaithful to work, C_5 = Not devoted to work or duty, C_6 = Not Punctual, C_7 = Misbehaviours, C_8 = Irregular to duty or work, C_9 = Not attachment to work, C_{10} = Lateness to duty, C_{11} = Non Challant attitude to work, C_{12} = Inconsistency, C_{13} = Stealing of Government funds and so on.

The constant K belongs to all the above characters which is known as the constant of proportionality. The constant K is known as the Mathematical Effected Virus Constant. Note that the E – virus constant K has main role for strongly increasing the corruption. Based on the values of the constant K, we have three types of corruption. They are:

Negative Corruption: It is the quantity in which the value of K is less than zero. It is very useful for the health and growth of the society and National Development.

Constant Corruption: It is the quantity in which the value of K is equal to zero. When $K = 0$, then the health of the society in the country or world will become strong.

Positive Corruption: It is the quantity in which the value of K is greater than zero. If it happens, then the health of the society in the country or world will become very weak or sick and hence, there will be poor growth of National Development.

Suppose the rate of growth of corruption at any time t ($\frac{dC}{dt}$) is proportional to the number of corruption practices (C) at time t , then, the growth law is given as

$$\frac{dC}{dt} \propto C, \quad C(0) = C_0 \tag{2}$$

$$\frac{dC}{dt} = KC \tag{3}$$

On solving equation (3), we obtain,

$$\ln C = Kt + \alpha \tag{4}$$

where K is the constant of proportionality and it denotes the characters of person which leads to increase in the corruption.

Equation (4) can be re written in the form;

$$C = Pe^{Kt} \text{ where } P = e^\alpha \tag{5}$$

$$\text{when } t = 0, C_0 = P \text{ i.e. } C = C_0e^{Kt}, \text{ for } K > 0 \tag{6}$$

Equation (6) is known as the Mathematical Corruption Growth Formula.

In addition, the rate of growth of corruptionpractices at any time t ($\frac{dC}{dt}$) is inversely proportional to the National Development

(ND) of a country which is given in the form;

$$\frac{dC}{dt} \propto \frac{1}{ND} \tag{7}$$

Therefore, equation (7) is governed by the differential equation

$$\frac{dC}{dt} = \frac{K}{ND}, \text{ subject to the condition } C(0) = C_0 \tag{8}$$

Solving equation (8) using the separation of variables and integrating both sides, we obtain,

$$NDC = Kt + \alpha \tag{9}$$

Combining equations(4) and (9), we obtain,

$$NDC = \ln C \text{ i.e. } Kt + \alpha = \ln C, \tag{10}$$

Therefore,

$$ND = \frac{\ln C}{C} \tag{11}$$

where α represents the measures to reduce the rate of growth of corruption practices in the society and K denotes the Mathematical E – virus constant i.e. negative character of persons which leads to increase in the rate of corruption.

Therefore, equation (11) gives the mathematical modelthat shows the relationship between Corruption Practices and National Development.

Suppose we assume that there was no corruption practice in the country before 1st October, 1960 (Independence Day) with a population of 45.2 Million at that time and the Mathematical E-virus value K is increasing by 10% every year with $C_0 = 4.52$ (10% of 45.2 Million), then we obtain the Table 1 below.

Table 1: Corruption and National Development in Millions

Time (years)	K	Corruption (C)	National Development (ND = ln C/C)
1 st October, 1960	0.1	4.52	0.3337415916
1 st October, 1970	0.2	5.52074	0.3094715399
1 st October, 1980	0.3	8.23598	0.2560123216
1 st October, 1990	0.4	15.00693	0.1804840893
1 st October, 2000	0.5	33.39853	0.1050498895
1 st October, 2010	0.6	90.78663	0.04966052852
1 st October, 2017	0.7	244.328099	0.02250462396
1 st October, 2020	0.8	549.227087	0.01148616327
1 st October, 2030	0.9	2461.465034	0.003172302627
1 st October, 2040	1.0	13473.9301	0.0007056969958

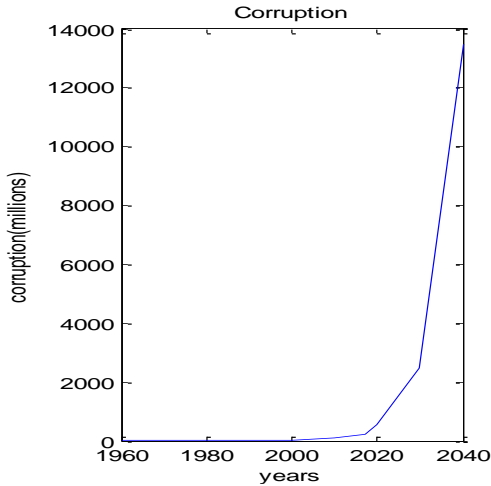


Fig. 2: Corruption against Time (years)

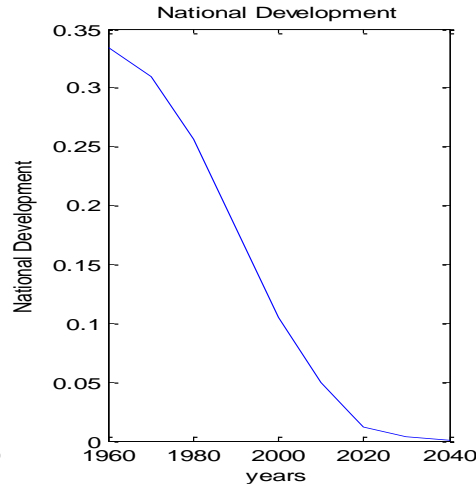


Fig. 3: National Development against Time (years)

Discussion

From the Table 1 and the diagrams in Fig. 2 and Fig. 3, they show that if the growth corruption practices increase, then the National Development in the country will decrease accordingly. The growth of corruption practices depends on the Mathematical E-virus constant K. When $K > 0$, the growth of positive corruption will increase to the high level and at that time developments in the society in any fields will decrease. When necessary measures are taken to reduce the corruption rate in the country, then the mathematical E – virus K will be less than 0 ($K < 0$) and at that time National Development of the society will increase in any fields of the country.

Conclusion

We have tried to highlight the pivotal roles of the mathematical modelling in sustaining National Development and we gave an illustration on the mathematical corruption model to show that as the rate of corruption increases with time, National Development decreases with time. When necessary measures are taken to reduce the rate of corruption in the country, then national development of the society will increase in any fields or areas of the country. Despite the paradoxical nature surrounding this subject, the conclusion suggests that corruption have a negative impact on efficiency, growth and welfare of the citizens in the country.

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