

Transient Stability Analysis of Delta IV Multi-machine Generating Station System Collapse: The Equal-Area Criterion Application

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Abstract

This paper analyzed the Delta IV multi-machine generating station transient stability to determine if the system was stable or not during system collapse. The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbances.

The transient disturbance was analyzed using the equal area criterion techniques to determine the values of synchronizing power coefficient, the critical clearing angle and the critical fault clearing time during system collapse. In this study, available data were obtained from the Transmission Company of Nigeria, Delta IV (Ughelli) Power Station. The transient stability was simulated using MATLAB Version R2012a, for a clearing time t_c of 0.5 second and t_c of 0.769 second respectively. The results show the initial power angle of 17.71° degree Celsius, maximum angle swing of 162.29° , critical clearing angle of 100.64° , and critical fault clearing time of 0.769 seconds. Simulation for a clearing time of 0.4 second, the system was found to be critically stable. But for a critical fault clearing time of 0.769 second, the curves showed that the system was unstable. The swing curves show that machines phase angle increases without limit. The longer the duration of the fault, the more the rotor accelerated and the larger the rotor angle becomes thus, the synchronous machine loses synchronism. The maximum fault clearing time for which the system remains unstable is the critical fault clearing time, and the corresponding angle is the critical fault clearing angle.

Keywords: Disturbance, Equal-area criterion, Stability, Swing Equation, Synchronous machine, Synchronism, Transient Stability.

1. Introduction

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. The stability of an interconnected power system is its ability to return to original equilibrium position on the occurrence of a disturbance or to another equilibrium state which is generally in the proximity of the initial equilibrium point. With interconnected systems continually growing in size and extending over vast geographical areas, it is becoming increasingly more difficult to maintain synchronism between various parts of the power system. Ideally, the loads must be fed at constant voltage and frequency at all times [1, 2]. The first requirement of reliable service is to keep the synchronous generators running in parallel and with adequate capacity to meet the load demand. Synchronous machines do not easily fall out of step under normal conditions. If a machine tends to speed up or slow down, synchronizing forces tend to keep it in step. Conditions do arise, however, such as a fault on the network, failure in a piece of equipment, sudden application of a major load such as a steel mill, or loss of a line or generating unit, in which operation is such that the synchronizing forces for one or more machines cannot be adequate, and small impacts in the system can cause these machines to lose synchronism. Secondly, is to maintain the integrity of the power network. The high-voltage transmission system connects the generating stations and the load centers.

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Interruptions in this network may hinder the flow of power to the load. This usually requires a study of large geographical areas since almost all power systems are interconnected with neighboring systems. Random changes in load are taking place at all times, with subsequent adjustments of generation [3, 4]. Synchronism frequently can be lost in that transition period, or growing oscillations can occur over a transmission line, eventually leading to its tripping. A method known as the equal area criterion can be used for quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance. The method is applicable to one or two-machine system connected to an infinite bus. The application of the equal area criterion is to determine if the system is stable or not after a given contingency and to investigate the stability of the system [5]. In normal steady state operation all synchronous machines in the system rotate with the same electrical angular velocity, but as a consequence of disturbances one or more generators can be accelerated or decelerated and there is risk that they can lose synchronism. This can have a large impact on system stability and generators losing synchronism must be disconnected otherwise they could cause severe damaged [5, 6].

2.0 GENERAL STUDY OF THE SWING EQUATION

2.1 STABILITY

The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium is known as stability. The problem of interest is where a power system operating under a steady load condition is perturbed, causing the readjustment of the voltage angles of the synchronous machines. If such an occurrence creates an unbalance between the system generation and load, it results in the establishment of a new steady-state operating condition, with the subsequent adjustment of the voltage angles. The perturbation could be a major disturbance such as the loss of a generator, a fault or the loss of a line, or a combination of such events [6, 7]. It could also be a small load or random load changes occurring under normal operating conditions. Adjustment to the new operating condition is called the transient period. The system behavior during this time is called the dynamic system performance, which is of concern in defining system stability. The main criterion for stability is that the synchronous machines maintain synchronism at the end of the transient period [8]. So if the oscillatory response of a power system during the transient period following a disturbance is damped and the system settles in a finite time to a new steady operating condition, the system is stable. If the system is not stable, it is considered unstable. This definition of stability requires that the system oscillations be damped [9, 10]. This condition is called asymptotic stability and means that the system contains inherent forces that tend to reduce oscillations. This is a desirable feature in many systems and is considered necessary for power systems. This also excludes continuous oscillation from the family of stable systems, although oscillators are stable in a mathematical sense. The reason is practical since a continually oscillating system would be undesirable for both the supplier and the user of electric power [11, 12, 13]. For convenience of analysis, stability problems are generally divided into two major categories—steady state stability and transient state stability.

2.2 THE SWING EQUATION

Stability analysis of a power system is an extensive and complicated task. The synchronous machine models form the basis for the derivation of the swing equation describing the electro-mechanical oscillations in a power system. The per unit inertia constant H in MJ/MVA is the kinetic energy stored in the rotating parts of the machine at synchronous speed per unit Megavolt ampere (MVA) rating of the machine. Thus, if G is the MVA rating of the machine, then

$$GH = \frac{1}{2} J \omega_m^2 \times 10^{-6} \tag{1}$$

Where J = polar moment of inertia of the prime mover and generator in $kg\cdot m^2$ and ω_m is angular velocity (mech. Rad/sec.). If M is the corresponding angular momentum, then

$$M = J \omega_m \tag{2}$$

From equations (1) and (2) we have $GH = \frac{1}{2} M \omega_m$

$$\text{or } M = \frac{2GH}{\omega_m} \quad \text{MJ} - s / \text{mech. rad} \tag{2a}$$

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the power angle or torque angle. During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air gap magneto-motive-force (mmf), a relative motion begins. A generator receives mechanical torque input T_m via shaft from the prime mover and develops electromagnetic torque T_e running at synchronous speed ω_{syn} which opposes the mechanical torque T_m . The subscripts m and e denotes the mechanical quantity, and an electrical quantity. The algebraic difference between T_m and T_e is the net torque T_a which causes acceleration of rotor.

$$\text{That is } T_a = T_m - T_e \tag{3}$$

The different torques and powers of a synchronous machine is schematically depicted as shown in Figure 1.

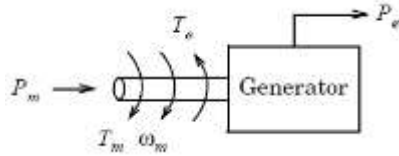


Figure 1: Schematic description of Powers and Torques in synchronous machines.

When the machine is operating in steady state T_a is zero and there is no acceleration. Whenever disturbances occur, such as a change in load or a fault, the mechanical power input P_m no longer equal the electrical power output P_e , it results in an accelerating or retardation depending on whether T_a is positive or negative.

If P_a is the corresponding accelerating power, then $P_a = P_m - P_e = M \frac{d^2\theta_m}{dt^2} + D \frac{d\theta_m}{dt}$ (4)

Where P_a is in megawatt, D is a damping coefficient, and θ_m is the mechanical angular position of the rotor.

In a steady state, $\frac{d\theta_m}{dt} = \omega_m = \text{constant}$, or $\theta_m = \omega_m t + \delta_m$ (5)

Where δ_m is the power angle of the synchronous machine in mechanical radians.

Neglecting damping and substituting (5) in (4) yields

$$M \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \tag{6}$$

Equation (6) is known as the swing equation [1-7].

The relations between electrical power angle δ and mechanical power angle δ_m and electrical speed and mechanical speed are $\delta = \frac{P}{2} \delta_m$, and $\omega = \frac{P}{2} \omega_m$, where P is the number of pole.

For a two or more machines system, the equation can be written as

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \tag{7a}$$

$$\text{and } M_2 \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \tag{7b}$$

Where subscripts 1 and 2 correspond to machines 1 and 2 respectively

Denoting the relative angle between the two rotor axis by δ , such that $\delta = \delta_1 - \delta_2$, then equations (7a) and (7b) can be combined and simplified to

$$M \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ for two machines} \tag{8}$$

Where $M = \frac{M_1 M_2}{M_1 + M_2}$ is the equivalent angular momentum of the machines.

If we combine equations (2a) and (6) and divide by G, we obtain the per unit swing equation as

$$\frac{2H}{\omega_m} \frac{d^2\delta_m}{dt^2} = P_m - P_e \text{ p.u} \tag{9}$$

Thus, two machines can be reduced to an equivalent one machine – infinite bus system. The equivalent swing equation (6) of the two machines can be written as

$$\frac{2H_1}{\omega_m} \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} = P_{a1} \tag{10a}$$

$$\frac{2H_2}{\omega_m} \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} = P_{a2} \tag{10b}$$

Equations (10a) and (10b) can be combined as

$$\frac{d^2\delta_{12}}{dt^2} = \frac{\omega_m P_{a1}}{2H_1} - \frac{\omega_m P_{a2}}{2H_2}, \text{ where } \delta_{12} = \delta_1 - \delta_2 \tag{11}$$

Equation (11) can be further written as

$$\frac{2}{\omega_m} \frac{H_1 H_2}{H_1 + H_2} \frac{d^2\delta_{12}}{dt^2} = \frac{H_2 P_{a1} - H_1 P_{a2}}{H_1 + H_2} \tag{12}$$

$$\text{Or } \frac{2H}{\omega_m} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \tag{13}$$

Where $H = \frac{H_1 H_2}{H_1 + H_2}$ is the equivalent inertia constant, and $\delta = \delta_{12}$

$$\text{The equivalent mechanical input } P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} \tag{14a}$$

$$\text{also } P_{max} = \frac{EV}{X} \sin\delta$$

$$\text{and the equivalent electrical output } P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2} \tag{14b}$$

The swing equation contains information regarding the machine dynamics and stability.

However, for multi-machine systems, the equation can be written similar to one-machine system by the following assumptions:

- a) Each synchronous machine is represented by a constant voltage source E behind the direct axis transient reactance X_d .
- b) Resistances of lines, transformers, and synchronous machines are neglected.
- c) The governor's actions are neglected and the input powers are assumed to remain constant during the entire period of simulation.
- d) The mechanical rotor angle of each machine coincides with the angle of the voltage behind the machine reactance.
- e) Using pre-fault bus voltages, all loads are converted to equivalent admittances to ground.
- f) Machines belong to the same station swing together and are said to be coherent, coherent machines are equivalent to one machine[8-11].

For a particular case where the electrical power P_e during fault is zero, the swing equation as given in equation (13), becomes

$$\frac{2H}{\omega_m} \frac{d^2\delta}{dt} = P_m \text{ or } \frac{d^2\delta}{dt} = \frac{\pi f_0}{H} P_m \tag{15}$$

Integrating both sides $\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt = \frac{\pi f_0}{H} P_m t$

Integrating again, we get $\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$ (16)

Applying equal area-criterion, we have

$$\int_{\delta_0}^{\delta_c} P_m d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max} \sin\delta - P_m) d\delta \tag{17}$$

Integrating both sides, we have $P_m (\delta_c - \delta_0) = P_{max} (\cos\delta_c - \cos\delta_{max}) - P_m (\delta_{max} - \delta_c)$

Solving for δ_c , we get $\cos\delta_c = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos\delta_{max}$ (18)

If we wish to determine the critical clearing angle δ_c , we note that the maximum allowable value of δ_1 the overshoot angle is δ_m . Should δ reach δ_m the accelerating power will again become positive and synchronism will be lost.

Since $\delta_m = \delta_{1max} = \pi - \delta_0$ we have that $\cos\delta_c = (\pi - 2\delta_0) \sin\delta_0 + \cos(\pi - \delta_0)$

Or $\delta_c = \cos^{-1}[\pi - 2\delta_0) \sin\delta_0 - \cos\delta_0]$.

Thus, if δ_c is the critical clearing angle, the corresponding critical clearing time is [7, 15]

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}} \tag{19}$$

The machine currents prior to disturbance are calculated from

$$I_i = \frac{S}{V} \quad i = 1, 2, \dots, m \tag{20}$$

Where m is the number of generators, V is the terminal voltage of the generator, and S is the apparent powers. The generators armature resistances are usually neglected and the voltages behind the transient reactance's are obtained [16] as

$$E_i^j = V_i + jX_d^j I_i \tag{21}$$

3.0 TRANSIENT STABILITY ANALYSIS USING THE EQUAL-AREA CRITERION

3.1 A Case Study of Delta IV Power System Collapse

The power system network of Delta IV generating station with the equivalent circuit of a three synchronous machines modeled with a constant electro-magnetic field through a transmission line having transient reactance X_d is shown in Figure 2.

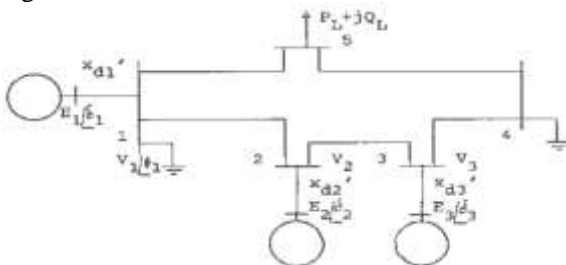


Figure 2: Three Synchronous machines connected to infinite bus

The E_1 , E_2 and E_3 are the voltages behind transient reactances of the generators. The reactances jx_{d1} , jx_{d2} and jx_{d3} are the equivalent reactance's including transformers and parallel lines. These groups of synchronous machines are connected to a larger system through more power lines. The angle of the electromagnetic field is assumed to coincide with the rotor angle. A three phase to earth fault occurs on Delta – Benin 330kV line between buses 1 and 5, and the backup protective distances relays tripped at both ends of the transmission line resulting power system outage. The power that was flowing on the tripped line was transferred to other lines, that is, the Benin line II and Sapele lines both operating at their peak capacity causing them to become overloaded. In this study, available data were obtained from the Transmission Company of Nigeria, Delta IV (ughelli) Power Station [17]. The data includes the generation schedule, machine data and load data for the regulated buses are shown in tables 1, 2, and 3, whose voltage specified as $V = 1 \angle 0$ pu, is taken as the slack bus. The generator's total transient reactance's and the inertia constants expressed on a 100 MVA base is shown in table 2. The real power transferred was 0.7 pu at 0.8 pf lagging. The objective of this study was to determine the values of synchronizing power coefficient, how fast the fault must be cleared to ensure stability of the system in terms of the critical fault clearing time and the critical clearing angle.

Table 1: Generation Schedule Table 3: Load Data

GENERATION SCHEDULE				LOAD DATA		
Bus No.	Voltage Mag.	Generation MW	Mvar Limits	Bus No.	LOAD	
			Max.		MW	Mvar
1	1.00	0	0	1	0	0
2	1.04	150	140	2	0	0
3	1.03	100	90	3	0	0
				4	100	70
				5	90	30
				6	160	110

Table 2: Machine Data

MACHINE DATA			
Gen	Ra	X'_d	H
1	0	0.20	15
2	0	0.15	6
3	0	0.25	7

4.0 Analysis, Results and Discussion

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This is as a result of loss of generation, loss of large load, or a fault on the system. The models and equations derived above are applied to the synchronous machines connected to an infinite bus. Useful and importance information about the behaviors and oscillations of the machines and the features of the equilibrium points of the system are determined from the analysis. The disturbance considered is a three phase to earth fault on one of the lines close to the generator.

From Table 1, the supply Voltage $V = 1 \angle 0$ pu

The per unit apparent power $S = \frac{P}{\cos \phi} = \frac{0.7}{0.8} \angle \cos^{-1} 0.8 = 0.875 \angle 36.87^\circ$ p.u.

From equation (20) the current flowing into the infinite bus is $I = \frac{S}{V} = \frac{0.875 \angle 36.87^\circ}{1 \angle 0} = 0.875 \angle 36.87^\circ$ p.u.

Also from equation (21) the transient internal Voltage

$$E = V + jXI = 1 \angle 0 + (0.6 \angle -36.87^\circ)(0.875 \angle 90^\circ) = 1 + j0 + 0.525 \angle 53.13^\circ = 1 + j0 + 0.315 + j0.42 = 1.315 + j0.42 = 1.38 \angle 17.71^\circ$$

Thus the initial operating angle $\delta_0 = 17.71^\circ = 0.309$ radian.

The synchronizing power coefficient $P_e = \frac{E_1 V_1}{X} \cos \delta_0 = \frac{1.38 \times 1}{0.6} \cos (17.71^\circ) = 2.191$ p.u.

The power angle equation after the fault is

$$P_{max} \sin \delta = \frac{1.38 \times 1}{0.6} \sin \delta = 2.3 \sin \delta$$

The initial operating angle is $2.3 \sin \delta_0 = 0.7$ or $\delta_0 = 17.71^\circ = 0.309$ rad

Maximum angle swing $\delta_{max} = 180^\circ - \delta_0 = 180^\circ - 17.71^\circ = 162.29^\circ = 2.832 \text{ rad}$ shown in figure 4

Since the fault occurred at the beginning of the transmission line, the power transfer during fault is zero. From equation (18) the critical clearing angle is

$$\cos \delta_c = \frac{0.7}{2.3} (2.832 - 0.309) + \cos 162.29 = (-0.1847)$$

Thus, the critical clearing angle $\delta_c = \cos^{-1}(-0.1847) = 100.64^\circ = 1.756 \text{ rad}$

From equation (14) and table 2, the equivalent inertia constant H of the machines becomes,

$$H = \frac{H_1 H_2 H_3}{H_1 + H_2 + H_3} = \frac{15 \times 6 \times 7}{15 + 6 + 7} = 22.5 \text{ MJ/MVA}$$

From equation (19) the critical clearing time is

$$t_c = \sqrt{\frac{2 \times 22.5 (1.756 - 0.309)}{\pi \times 50 \times 0.7}} = 0.769 \text{ sec.}$$

Thus the critical fault clearing time is 0.769 seconds. The maximum fault clearing time for which the system remains unstable is called critical fault clearing time, and the corresponding angle is called critical fault clearing angle.

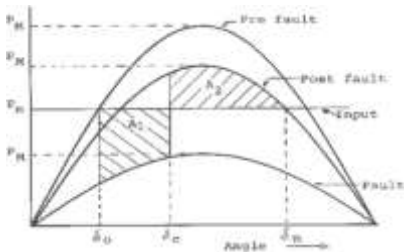


Figure 3: the electric power before and after the fault

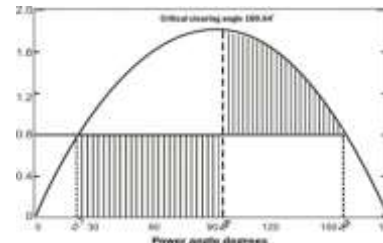


Figure 4: Equal area criterion applied to power angle curve

4.1 Results and Discussion

The power angle curves before and after the fault is plotted as shown in figure 3. The results of the analysis show that the generator is operating at the initial power angle δ_0 of 17.71° , maximum angle swing δ_{max} of 162.29° and new operating angle of 100.64° with a critical fault clearing time of 0.769 seconds using the equal area criterion as plotted shaded in figure 4. At $t = 0$, when the short circuit occurs, the electrical power output P_e instantaneously drop to zero and remains at zero during the fault since power cannot be transferred past faulted bus.

During the fault the rotor accelerated since $P_e = 0$, hence the mechanical power input $P_m - P_e > 0$, so the rotor accelerated and the system becomes unstable. Consequently, the generators fell out of step or losses synchronism. As shown in the figure, the system was stable for a fault clearing angle of up to the critical clearing angle δ_c , when the areas A_1 and A_2 are equal. For a clearing angle greater than δ_c the system became unstable. The corresponding maximum rotor angle reached is δ_m . Time domain simulations of the system show the principal behavior of stable and unstable solutions. The transient stability simulations conducted using MATLAB, Version R2012a for a clearing time t_c of 0.4 second and for calculated critical clearing time t_c of 0.769 second respectively are shown in figure 5. For a clearing time of 0.4 second it was found to be critically stable; but with the fault clearing time of 0.769 second, the system was unstable and losses synchronism. The swing curves shown in figure 6 indicate that machine 2 phase angle increases without limit. The generator cannot be decelerated enough before it reaches the critical point, but it passes this point and is further accelerated and loses synchronism. The swing curves shown that machine 2 phase angle increases without limit as indicated in Figure 6. Thus, the system was unstable and the generator fell out of step or loses synchronism.

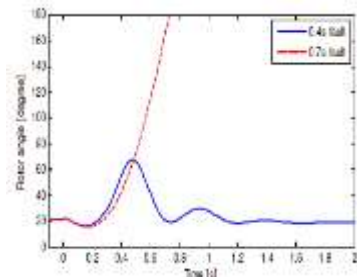


Figure 5. Transient stability fault cleared at 0.4 and 0.769 Sec

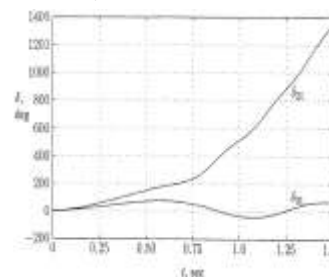


Figure 6: Phase angle difference for machines 21 and 31

Results further show that the longer the duration of the fault, the more the rotor will be accelerated and the larger the rotor angle become. If the duration of the fault is too long there is the risk that the rotor swing will be so large that it passes the right equilibrium point and the synchronous machine loses synchronism.

4.2 Damping Effects

In a stable system when the machine rotor reaches the maximum angle $d\delta/dt = 0$, and the net torque on the rotor retarded. The rotor thus start to swing backward and go past the post-fault equilibrium point and continue until it reaches the point where $d\delta/dt = 0$, and the total area between the post-fault power-angle curve and the input line is zero, i.e., the area above the input line is equal to the area below the input line as shown in figure 4. This represented the equal area criterion. Results shown that, including the effect of damping, the area between the power angle and the input line becomes smaller in each successive swing, thereby allowing the rotor to eventually settle at the post-fault equilibrium point. If the damping is retarded, the component of electrical power decreasing with increase in angle, the area between the power angle curve and the input line will increase in each subsequent swing, thereby causing instability. The difference between mechanical powers fed into the machine and the electrical output power cause a motion of the rotor relative to a rotation with constant angular velocity.

5.0 CONCLUSION

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. The transient stability of multi-machine synchronous generators using the equal area criterion application was presented and analyzed. The equal-area criterion application is to determine if a system is stable or not for a given data and disturbances. Transient stability simulation was conducted for a clearing time t_c of 0.4 second and t_c of 0.769 second respectively. It shows that the system was stable for a fault clearing angle of up to the critical clearing angle δ_c , when the areas A_1 and A_2 are equal. For a clearing angle greater than δ_c the system became unstable and the generator fell out of step or loses synchronism.

The effect of damping was presented; the area between the power angle and the input line becomes smaller in each successive swing, thereby allowing the rotor to eventually settle at the post-fault equilibrium point. If the damping is retarded, the component of electrical power decreasing with increase in angle, the area between the power angle curve and the input line will increase in each subsequent swing, thereby causing instability. Thus we saw that a two-machine system can be equivalently reduced to a one machine system connected to infinite bus bar. In order to improve system stability and reduce the possibility of loss of synchronism, the following factors should be considered. These include:

- a. Increase of the inertia constant of the generators. This makes the rotors more difficult to accelerate in connection with faults and the risk for losing synchronism is reduced.
- b. Reducing the magnitude of disturbance: large disturbances are typically due to faults which are cleared by protection relays. Reducing the duration of fault prevents loss of synchronism.
- c. Increase of system voltage. This increases the maximum electrical output P_e and for given maximum power P_m the stability margin is increased.
- d. Reduction of the transfer reactance X_e . This will also increase the maximum electrical output P_e as in the previous case. This can be achieved by constructing parallel lines, or by installing series capacitors on existing lines or new lines. By installing series capacitors the effective reactance of the line is reduced.
- e. Installation of fast protections and fast breakers. In this way the time with a fault connected can be reduced and thereby the time during which the generator rotors are accelerated.

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