Journal of the Nigerian Association of Mathematical Physics
Volume 46 **(May,** ***2018 Issue*****), pp101 –106**
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SENSITIVITY OF PRIOR INFORMATION ON THE ESTIMATION OF HETEROGENEOUS DYNAMIC MICRO PANEL MODEL

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Abstract

The key feature of Bayesian estimation is typically based on sensitivity of choice of prior which determines the inference made about posterior estimates. The prior density and the likelihood function are crucial elements of any analysis, and both must be fully specified for estimation to proceed. This paper extends the importance of prior information under estimation of Bayesian approach to heterogeneous panel data models with lagged dependent variable. It considers the fundamental issues of statistical inference of a random coefficients formulation using Bayesian approach. Noninformative and Informative priors are considered in this study and posterior results are based on 10,000 replications with N = 20 and T = 5.

Theoretical findings are accompanied by extensive Markov Chain Monte Carlo experiments, which show that the estimator perform well so long as the dimension of N>T. The results show closed (estimates)/values of the parameters which establish the sensitivity of prior in the estimation.

Keywords**:** Heterogeneous effect, Dynamic panel data, Hierarchical Bayesian Inference, Informative and Noninformative prior, Posterior Simulation (Gibb sampling)

1.0 Introduction

Panel data is a special case of longitudinal data in which some units of observation such as individuals, firms, or nations, are followed over a numbers of time periods. Panel data analysis plays an important role in modern econometric methodology, because it is often possible to take advantage of the grouping structure to address substantive economic questions more completely than is possible with simpler forms of data. In particular, the grouping structure can be used to estimate models with complicated forms of heterogeneity across units. For panel data studies with large N and small T, it is usual to pool the observations, assuming homogeneity of the slope coefficients. The latter is a testable assumption which is quite often rejected, Moreover, with the increasing time dimension of panel data sets, some researchers including [1, 2] have questioned the poolability of the data across heterogeneous units. Instead, they argue in favour of heterogeneous estimates that can be combined to obtain homogeneous estimates if the need arises. To buttress this point, Robertson and Symons [1] studied the properties of some panel data estimators when the regression coefficients vary across individuals, that is, when they are *heterogeneous* but are assumed *homogeneous* in estimation. This is done for both stationary and nonstationary regressors. The basic conclusion is that severe biases can occur in dynamic estimation even for relatively small parameter variation [3,4].

However, working with panel data requires care to ensure that the techniques used are appropriate to the problem at hand. With heterogeneity, an additional unit of data is at best only partially informative about the model parameters common to all units. Panel data sets are also more effective in identifying and estimating effects that are simply not detectable in pure cross-sectional or pure time series data. In particular, panel data sets are more effective in studying complex issues of dynamic behaviour. The dynamic relationships are characterized by the presence of a lagged dependent variable among the regressors. Traditionally, the econometrics literature has focused on models that allow intercepts to vary across units but that assume the same autoregressive coefficients for all units [5, 6, 7]. Recent literature on large dynamic panels focuses mostly on how to deal with cross-sectional (CS) dependence assuming slope homogeneity. Estimation of panel data models with lagged dependent variables and cross-sectionally dependent errors has been considered in [8], who proposed a Gaussian quasi maximum likelihood estimator (QMLE). Moon and Weidner analysis assumes homogeneous coefficients, and therefore it is not applicable to dynamic panels with unobserved individual specific effects. Similarly, the interactive-effects estimator (IFE) developed by [11] also allow for cross-sectionally dependent errors, but assume homogeneous slopes. Song [12] extends the analysis of [11] by allowing for a lagged dependent variable as well as coefficient heterogeneity, but provides results on the estimation of cross-section specific coefficients only.

In many empirical settings in economics, Bayesian methods appear statistically more appropriate, and computationally more attractive, than the classical or frequentist methods typically used. Recent textbooks discussing modern Bayesian methods with an applied focus include [11,13,14].

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A conventional Bayesian analysis does require a fully specified parameter model, as well as a prior distribution on all the parameters of the model. Bayesian methods are especially attractive in settings with many parameters.
The *prior distribution* is a key part of Bayesian inference and represents the information about an uncertain parameter that is combined with the probability distribution of new data to yield the **posterior distribution**, which in turn is used for future inferences and decisions involving parameter. There are two types of priors; (1) noninformative and (2) informative priors, a non-informative prior signify ignorance or not in possession of enough information about the parameters of the model to aid in drawing posterior inferences, while informative prior summarizes what you know about parameters before seeing the data. It contains any non-data information about the parameter of interest when there is sufficient prior information on the shape and scale of the distribution of a model parameter that can be systematically incorporated into the prior distribution [5, 12,15].
The advantage of a Bayesian approach is that prior information can systematically be included in the analysis. The influence of the prior depends mainly on its precision and on the variance of the variables. The higher the precision of the prior distribution, the bigger its influence [16, 17]
However, econometrics is a public science where empirical results are presented to a wide variety of readers. In many cases, most readers may be able to agree on what a sensible prior might be. In the case where different researchers can approach a problem with very different priors, a Bayesian analysis with only a single prior can be criticized. A *prior sensitivity analysis* can be carried out to proffer solution. This means that empirical results can be presented using various priors. If empirical results are basically the same for various sensible priors, then the reader is reassured that researchers with different beliefs can, after looking at the data, come to agreement [18].
Bayesian methods are especially attractive in settings with many parameters. Examples discussed in this paper include dynamic panel data with individual-level heterogeneity in multiple parameters. In such settings, methods that attempt to estimate every parameter precisely without linking it to similar parameters, often have poor repeated sampling properties. This shows up in Bayesian analyses in the dogmatic posterior distributions resulting from flat prior distributions. A more attractive approach that is successfully applied in the aforementioned examples can be based on hierarchical prior distributions where the parameters are assumed to be drawn independently from a common distribution with unknown location and scale. The recent computational advances make such models feasible in many settings [19, 20, 21].
This study focuses on the sensitivity of prior distribution on posterior estimate of a dynamic individual heterogeneity panel data model.
The rest of the paper is organized as follows. Section 2 describes the materials and methods of the random coefficient dynamic panel data model; section 3 describes the hierarchical prior distribution for the model parameters Section 4 provides Data Simulation techniques for the experiment. The analysis and results are discussed in section 5 while section 6 concludes the study.

2.0 Materials and Methods

2.1 The Model

Consider a general form of dynamic random coefficient panel data model:

$$y_{it} = \omega_i y_{i,t-1} + \sum_{k=0}^{K} X_{kit}\theta_{kit} + \mu_{it} \quad (1)$$

Assume that parameters do not vary with time (the time stability issue is not specific to panel data econometrics [25] that is $\theta_{kit} = \theta_{ki}$. Therefore,

$$y_{it} = \omega_i y_{i,t-1} + \sum_{k=0}^{K} X_{kit}\theta_{ki} + \mu_{it} \quad i = 1,...,N \text{ and } t = 1,...,T\text{, k =0,1,2}$$

That is,

$$y_{it} = \omega_i y_{i,t-1} + \theta_{0i} + X_{1it}\theta_{1i} + X_{2it}\theta_{2i} + \mu_{it}$$

Now,

$$y_{it} = z_{it}\lambda_i + \mu_{it} \quad (2)$$

Where

$$z_{it} = (x_{it} y_{i,t-1})', \lambda_i = (\theta_i' \omega_i)'$$

y_{it} is the response variable of interest, where the indices i and t (i = 1,...,n, t = 1,...,T) refer to individual i and time period t, respectively. z_{it} is the unit specific regressors, λ_i is the regression coefficients while λ_i is IID $(\lambda_i, \sigma_\lambda^2)$. Also μ_{it} is the disturbance, or error term, that has well-defined probabilistic properties. The objective in this case is to obtain consistent estimates of the mean values of λ_i .

2.1 Bayesian Conjugate Linear Regression Model Estimation

Bayesian methods focus on five essential elements. First, the incorporation of prior information. Prior information is generally specified quantitatively in the form of a distribution(e.g. normal-gamma distribution) and represents a probability distribution for a coefficient; meaning, the distribution of probable values for a coefficient we are attempting to model. Second, the prior is combined with a likelihood function. The likelihood function represents data. Third, the combination of the prior with the likelihood function results in the creation of a posterior distribution of coefficient values. Fourth, simulates are drawn from the posterior distribution of likely values for the population parameter. Fifth, basic statistics are used to summarize the empirical distribution of simulates from the posterior [5].

Likelihood, Prior and Posterior

Suppose that $y = y_1 \ . \ . \ . \ , y_n$ follows a distribution in a family parameterized by ($\lambda_i, \ \sigma_\lambda^2$) having a density $p(y \mid \lambda_i, \ \sigma_\lambda^2)$. It is noted from the linear regression model in (2) that the error term is $\mu_i \sim N(0, \sigma^2 I_n)$. According to the convention in Bayesian statistics, the normal

distribution is given as N~(mean, precision) instead of N~(mean, variance) [20]
Therefore, $y_i \sim N(z_i\hat{\lambda}_i,\ h^{-1}I_n)$ having a regression mean is $z_i\lambda_i$ and error precision as $h=\frac{1}{\sigma^2}$ with random variable y_i as the data information.
The expression for the likelihood density denoted by $p(y_i \mid \lambda_i, h)$ can be given as:

$$p(y_i \mid \lambda_i, h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}\exp[-\frac{h}{2}(y_i - z_i\lambda_i)'(y_i - z_i\lambda_i)] \qquad (3)$$

The prior of the individual effect λ_i is assumed to follow a normal distribution by the expression of likelihood function of a multivariate normal and the error precision follows a gamma distribution.

$$P(y/\lambda_i,\sigma^2) = \frac{1}{(2\pi)^{N/2}}\left(\{h^{N/2}\exp[-\frac{h}{2}(\lambda_i-\hat{\lambda}_i)'z_i'z_i(\lambda_i-\hat{\lambda}_i)]\}\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\}\right) \qquad (4)$$

The normal linear regression model defined in (2) depends upon the parameters λ_i and h it uses a natural conjugate prior where $p(\lambda_i \mid h)$ is a normal density and $p(h)$ a Gamma density. But here, we use a similar prior which assumes prior independence between λ_i and h In particular, we assume $p(\lambda_i, h) = p(\lambda_i)p(h)$ with $p(\lambda_i)$ being Normal and $p(h)$ being Gamma.
Thus,

$$p(\lambda_i) = \frac{1}{(2\pi)^{\frac{N}{2}}}|V_i|^{-\frac{1}{2}}\exp[-\frac{1}{2}(\lambda_i-\underline{\lambda}_i)'\underline{V}_i^{-1}(\lambda_i-\underline{\lambda}_i)] \text{ and}$$

$$P(h) = C_G^{-1}h^{\frac{v-2}{2}}\exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \qquad (5)$$

Where $\underline{\lambda}_i$ is the prior mean of λ_i and $Var(\lambda_i \mid h) = \underline{V}_i$ is the prior covariance matrix of λ_i with the parameters of Gamma distribution of h ($\underline{s}^{-2}$ and $\underline{v}$).
The posterior is proportional to the prior times the likelihood, which is also the information obtained after seeing the data, it can be a conjugate or independent or not taking a familiar distribution form, usually denoted $p(\lambda_i, h \mid y)$
The joint posterior can be written in the form below:

$$p(\lambda_i, h \mid y) \propto \exp[-\frac{1}{2}\{h(y_i-\lambda_i z_i)'(y_i-\lambda_i z_i)+(\lambda_i-\underline{\lambda}_i)'\underline{V}_i^{-1}(\lambda_i-\underline{\lambda}_i)\}].h^{\frac{N+v-2}{2}}\exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \qquad (6)$$

This joint posterior density for λ_i and h does not take any well-known distributional form; so it cannot be solved analytically but only through a posterior simulation method. Specifically, Gibbs sampling will be used to perform the Bayesian inference due to its desirable results that iterative sampling from the conditional distributions will lead to a sequence of random variables converging to the joint distribution.
Fully-Bayesian analyses of hierarchical linear models have been considered for at least forty years[4, 22, 23] and have remained a topic of theoretical and applied interest [4, 24, 25, 26]. Here, we explore the principles of hierarchical prior distributions in the context of a dynamic panel data model in order to examine the influence of the chosen prior distribution to on posterior distribution.

3.0 The Hierarchical Prior Distribution for the Model Parameters

It is assumed that λ_i are independently drawn from Normal distribution and thus, $\lambda_i \sim N(\varepsilon_\lambda, V_\lambda)$
The hierarchical structure of the prior arises if we treat ε_λ and V_λ as unknown parameters which require their own prior.
Where,

$$\varepsilon_\lambda / y,\lambda,h,V_\lambda \sim N(\bar{\varepsilon}_\lambda,\ \bar{\Sigma}_\lambda) \text{ and } V_\lambda^{-1}/y,\lambda,h,V_\lambda,\varepsilon_\lambda \sim W(\bar{v}_\lambda,\left[\bar{v}_\lambda\ \bar{V}_\lambda\right]^{-1})$$

The conditional posteriors for the $\lambda_i's$ are independent of one another, for $i = 1,...,N$, with

$$\lambda_i / y,h,\varepsilon_\lambda,V_\lambda \sim N(\bar{\lambda}_i,\bar{V}_i) \qquad (7)$$

Where

$$\bar{V}_i = (hz_i'z_i + V_\lambda^{-1})' \text{ and } \bar{\lambda}_i = \bar{V}_i(hz_i'y_i + V_\lambda^{-1}\varepsilon_\lambda)$$

The posterior conditional for the error precision has the form:

$$h/y,\lambda,V_\lambda,\varepsilon_\lambda \sim G(\bar{s}^{-2},\bar{v}) \qquad (8)$$

This Bayesian econometrics model involving the hierarchical prior and the error precision expressions above requires only random number generation from the Normal, Gamma and Wishart distributions.

4.0 Data Simulation

To observe the behaviour of the posterior distribution both informative and non-informative prior were considered. All the prior distribution of λ_i coefficients is identical at each hierarchical prior of the model. In order to compare the effect of differently precise prior information; the precision of the λ_i parameters priors are different between the models for the influence of the data to be examined.

The explanatory variables (X_{1it}, X_{2it}) are generated using uniform distribution (0, 1) that is $X_{it} \cup (0,1)$. The simulated lambda parameters are $\omega \sim B(0,1)$, $\theta_0 \sim N(0,0.25)$, $\theta_{1i} = 2$ and $\theta_{2i} = 3$. We generate values for the errors, $\mu_i \sim N(0, h^{-1})$ and use the independent variables and errors to generate the dependent variable, while prior hyperparameters are stated as

$\underline{\lambda}_4 = \varepsilon_\lambda = 0_4$, $\underline{\Sigma}_\lambda = 1$, $\underline{V}_\lambda^{-1} = 1$, and $\underline{v}_\lambda = 2$ for non-informative and $\underline{\lambda}_4 = \varepsilon_\lambda = (0.5, 0.5, 0.5, 0.5)$, $\underline{\Sigma}_\lambda = 0.05$, $\underline{V}_\lambda^{-1} = 0.05$, and $\underline{v}_\lambda = 10$ for informative prior. To examine the prior sensitivity on the posterior distribution, we set the error precision (h=0.04, 0.03, 0.02, 0.01) for noninformative and (h=25, 30, 50, 70) for informative prior.

The value of N (individual) is chosen to be 20 and time T=5 for the two sets of priors. Posterior results for the model are based on 10000 replications, with 1000 burn-in replications discarded and 9000 replications retained. MCMC diagnostics indicate convergence of all the Gibbs samplers and numerical standard errors indicate an approximation error which is small relative to posterior standard deviations of all parameters.

5.0 The Analysis and Results

Table 1: Posterior means and numerical standard errors (inbracket); when N = 20, T = 5

Posterior Estimate		
λ	Noninformative prior	Informative prior
	$\lambda \sim N(0, 25), h=0.04$	$\lambda \sim N(0.5, 0.04), h=25$
$\omega \sim B(0,1)$	0.14828 (0.00065)	0.14779 (0.00064)
$\theta_0 \sim N(0,0.25)$	0.87469 (0.00251)	0.86685 (0.00257)
θ (2)	1.54003 (0.00233)	1.54183 (0.00231)
θ (3)	2.23406 (0.00267)	2.23584 (0.00263)
	$\lambda \sim N(0, 30), h=0.03$	$\lambda \sim N(0.5, 0.03), h=30$
$\omega \sim B(0,1)$	0.14817 (0.00059)	0.14841 (0.00060)
$\theta_0 \sim N(0,0.25)$	0.87438 (0.00229)	0.87298 (0.00229)
θ (2)	1.54043 (0.00213)	1.53992 (0.00213)
θ (3)	2.23449 (0.00244)	2.23363 (0.00243)
	$\lambda \sim N(0, 50), h=0.02$	$\lambda \sim N(0.5, 0.02), h=50$
$\omega \sim B(0,1)$	0.14793 (0.00046)	0.14752 (0.00045)
$\theta_0 \sim N(0,0.25)$	0.87368 (0.00177)	0.86848 (0.00182)
θ (2)	1.54129 (0.00165)	1.54270 (0.00164)
θ (3)	2.23547 (0.00188)	2.23696 (0.00186)
	$\lambda \sim N(0, 70), h=0.01$	$\lambda \sim N(0.5, 0.01), h=70$
$\omega \sim B(0,1)$	0.14734 (0.00038)	0.14744 (0.00039)
$\theta_0 \sim N(0,0.25)$	0.86966 (0.00153)	0.86905 (0.00154)
θ (2)	1.54315 (0.00139)	1.54293 (0.00139)
θ (3)	2.23767 (0.00157)	2.23729 (0.00157)

Note: This Table presents the first stage of hierarchical prior $\varepsilon_\lambda / y, \lambda, h, V_\lambda \sim N(\bar{\varepsilon}_\lambda, \bar{\Sigma}_\lambda)$

Table 2: Posterior means and standard deviations (in bracket): When N=20, $\theta_0 \sim N(0, 0.25)$

λ	Non informative prior $\lambda \sim N(0, 25), h=0.04$				Informative prior $\lambda \sim N(0, 0.04), h=25$			
	$\omega \sim B(0,1)$	θ_0	θ_1 (2)	θ_2 (3)	$\omega \sim B(0,1)$	θ_0	θ_1	θ_2
1	0.14052 (0.0035)	0.88116 (0.0064)	1.48663 (0.0768)	2.35474 (0.1130)	0.14039 (0.0077)	0.79772 (0.0579)	1.65594 (0.1035)	2.19591 (0.0210)
2	0.09892 (0.04509)	0.95129 (0.0765)	1.76378 (0.2003)	2.31406 (0.0723)	0.16700 (0.0189)	0.70915 (0.1466)	1.53602 (0.0164)	2.15033 (0.0666)
3	0.12482 (0.0192)	0.96279 (0.0880)	1.63426 (0.0708)	2.27781 (0.0361)	0.17317 (0.0251)	0.74349 (0.1122)	1.55046 (0.0019)	2.09118 (0.1257)
4	0.15130 (0.0073)	0.90686 (0.0321)	1.59088 (0.0273)	2.16731 (0.0744)	0.14813 (0.0000)	0.96022 (0.1045)	1.55841 (0.0060)	2.15422 (0.0627)
5	0.12067 (0.0234)	0.85697 (0.0178)	1.62664 (0.0632)	2.29567 (0.0539)	0.13394 (0.0142)	0.89659 (0.0409)	1.51130 (0.0411)	2.28619 (0.0693)
6	0.20268 (0.0587)	0.80078 (0.0739)	1.49368 (0.0698)	1.99736 (0.2444)	0.15253 (0.0044)	0.76674 (0.0889)	1.58559 (0.0332)	2.19311 (0.0239)
7	0.11991 (0.0241)	0.93816 (0.0634)	1.57039 (0.0069)	2.34961 (0.1079)	0.13267 (0.0154)	0.83395 (0.0218)	1.65969 (0.1073)	2.24899 (0.0320)
8	0.14560 (0.0016)	0.78486 (0.0899)	1.49223 (0.0713)	2.32289 (0.0812)	0.15959 (0.0115))	0.82178 (0.0339)	1.50733 (0.0451)	2.21471 (0.0022)
9	0.12361 (0.0204)	0.94866 (0.0789)	1.57771 (0.0142)	2.29193 (0.0502)	0.15146 (0.0034)	0.83202 (0.0237)	1.47379 (0.0786)	2.25119 (0.0343)
10	0.16023 (0.0162)	0.74768 (0.1271)	1.50084 (0.0626)	2.28037 (0.0386)	0.12195 (0.0262)	0.83793 (0.0177)	1.57512 (0.0227)	2.29447 (0.0776)
11	0.16049 (0.0164)	0.94168 (0.0669)	1.47768 (0.0857)	2.16152 (0.0802)	0.17515 (0.02705)	0.89285 (0.0372)	1.44786 (0.0105)	2.16725 (0.0496)
12	0.17687 (0.0329)	0.72698 (0.1478)	1.46649 (0.0969)	2.14393 (0.0978)	0.13996 (0.0081)	0.78347 (0.0722)	1.58815 (0.0357)	2.17778 (0.0391)
13	0.11798 (0.0260)	1.02155 (0.1467)	1.72106 (0.1576)	2.24232 (0.0006)	0.10829 (0.0398)	0.81826 (0.0374)	1.74515 (0.1928)	2.27164 (0.0547)
14	0.12790 (0.0161)	0.91651 (0.0417)	1.58667 (0.0232)	2.31146 (0.0697)	0.15423 (0.0061)	0.91803 (0.0623)	1.45113 (0.1013)	2.29634 (0.0794)
15	0.15460 (0.0106)	0.94561 (0.0708)	1.55673 (0.0068)	2.17110 (0.0706)	0.18208 (0.0398)	0.90709 (0.0514)	1.39879 (0.1536)	2.17223 (0.0448)
16	0.18078 (0.0367)	0.78138 (0.0934)	1.49518 (0.0683)	2.13104 (0.1107)	0.13300 (0.0151)	0.92002 (0.0643)	1.59531 (0.0429)	2.28552 (0.0686)
17	0.14147 (0.0025)	0.72262 (0.1521)	1.63044 (0.0669)	2.22624 (0.0155)	0.16007 (0.0120)	0.90573 (0.0500)	1.52946 (0.0229)	2.14572 (0.0712)
18	0.13236 (0.0117)	0.98677 (0.1119)	1.51299 (0.0505)	2.32263 (0.1009)	0.15091 (0.0028)	0.95643 (0.1007)	1.51762 (0.0348)	2.22404 (0.0071)
19	0.18804 (0.0440)	0.73610 (0.1386)	1.46437 (0.0991)	2.10529 (0.1364)	0.12717 (0.0209)	0.91517 (0.0595)	1.61197 (0.0595)	2.29941 (0.0824)
20	0.11159 (0.0324)	0.93709 (0.0623)	1.62096 (0.0574)	2.34747 (0.1057)	0.15026 (0.0022)	0.89749 (0.0417)	1.54874 (0.0037)	2.21815 (0.0012)

Table 3: Posterior means and standard deviations (in bracket); When N=20, T=5, $\theta_0 \sim N(0, 0.25)$

λ	Noninformative prior $\lambda \sim N(0,30), h=0.03$				Informative prior $\lambda \sim N(0,0.03), h=30$			
	$\omega \sim B(0,1)$	θ_0	θ_1(2)	θ_2(3)	$\omega \sim B(0,1)$	θ_0	θ_1	θ_2
1	0.13515 (0.0101)	0.74574 (0.1179)	1.65266 (0.1091)	2.25992 (0.0015)	0.14087 (0.0070)	0.80456 (0.0529)	1.64635 (0.0946)	2.20009 (0.0190)
2	0.12988 (0.0154)	0.98236 (0.1186)	1.56066 (0.0171)	2.35729 (0.0989)	0.16516 (0.0172)	0.72357 (0.1339)	1.53688 (0.0148)	2.15854 (0.0605)
3	0.15873 (0.0134)	0.85927 (0.0045)	1.55589 (0.0123)	2.16784 (0.0905)	0.17079 (0.0229)	0.75484 (0.1027)	1.55009 (0.0016)	2.10447 (0.1146)
4	0.18393 (0.0387)	0.89867 (0.0349)	1.42942 (0.1141)	2.13133 (0.1270)	0.14800 (0.0000)	0.95291 (0.0953)	1.55726 (0.0055)	2.16167 (0.0574)
5	0.13931 (0.0059)	0.92575 (0.0620)	1.49919 (0.0443)	2.35728 (0.0989)	0.13504 (0.0129)	0.89498 (0.0374)	1.51418 (0.0375)	2.28227 (0.0631)
6	0.14315 (0.0021)	0.81746 (0.0463)	1.51804 (0.0255)	2.21531 (0.0431)	0.15196 (0.0040)	0.77625 (0.0813)	1.58211 (0.0304)	2.19748 (0.0216)
7	0.12321 (0.0221)	0.94967 (0.0859)	1.48616 (0.0574)	2.41259 (0.1542)	0.13384 (0.0141)	0.83773 (0.0198)	1.64974 (0.0979)	2.24849 (0.0294)
8	0.13122 (0.0140)	0.88272 (0.0189)	1.55729 (0.0137)	2.24916 (0.0092)	0.15844 (0.0104)	0.82655 (0.0310)	1.51062 (0.0411)	2.21712 (0.0019)
9	0.13431 (0.0109)	0.88272 (0.0184)	1.55729 (0.0128)	2.24916 (0.0619)	0.15101 (0.0031)	0.83595 (0.0216)	1.47996 (0.0717)	2.25039 (0.0313)
10	0.15444 (0.0092)	0.83841 (0.0253)	1.60232 (0.0588)	2.17335 (0.0850)	0.12407 (0.0239)	0.84142 (0.0161)	1.57247 (0.0207)	2.28997 (0.0709)
11	0.15158 (0.0063)	0.78690 (0.0768)	1.52948 (0.0141)	2.26987 (0.0115)	0.17267 (0.0247)	0.89139 (0.0339)	1.45631 (0.0954)	2.17362 (0.0454)
12	0.16991 (0.0246)	0.82857 (0.0351)	1.47315 (0.0704)	2.22646 (0.0320)	0.14049 (0.0075)	0.79151 (0.0660)	1.58446 (0.0327)	2.18351 (0.0355)
13	0.12529 (0.0199)	0.87823 (0.0145)	1.61488 (0.0713)	2.34047 (0.0821)	0.11156 (0.0363)	0.82345 (0.0341)	1.72776 (0.1760)	2.26925 (0.0502)
14	0.14879 (0.0035)	0.80068 (0.0630)	1.47597 (0.0676)	2.25101 (0.0074)	0.15359 (0.0056)	0.91457 (0.0570)	1.45922 (0.0925)	2.29146 (0.0723)
15	0.13657 (0.0087)	0.92966 (0.0659)	1.54692 (0.0034)	2.35194 (0.0936)	0.17901 (0.0311)	0.90438 (0.0468)	1.41150 (0.1403)	2.17812 (0.0409)
16	0.12919 (0.0161)	0.76727 (0.0964)	1.61961 (0.0761)	2.25065 (0.0077)	0.13418 (0.0137)	0.91639 (0.0588)	1.59089 (0.0391)	2.28167 (0.0625)
17	0.16401 (0.0187)	0.92604 (0.0623)	1.48861 (0.0549)	2.21805 (0.0403)	0.15889 (0.0109)	0.90313 (0.0456)	1.53084 (0.0209)	2.15399 (0.0651)
18	0.14527 (0.0000)	0.80211 (0.0616)	1.62246 (0.0789)	2.23760 (0.0207)	0.15056 (0.0026)	0.94955 (0.0920)	1.51997 (0.0543)	2.22542 (0.0063)
19	0.13475 (0.0105)	0.91222 (0.0485)	1.53457 (0.0089)	2.25869 (0.0003)	0.12885 (0.0191)	0.91198 (0.0544)	1.60609 (0.0543)	2.29437 (0.0752)
20	0.16656 (0.0213)	0.89734 (0.0336)	1.54741 (0.0039)	2.11841 (0.1399)	0.14994 (0.0019)	0.89572 (0.0381)	1.54840 (0.0034)	2.22015 (0.0011)

Note: Tables 2 and 3 represent the second stage of hierarchical prior $\lambda_i / y, h, \varepsilon_\lambda, V_\lambda \sim N(\bar{\lambda}_i, \bar{V}_i)$

6.0 CONCLUSION

A research models commonly used in classical approach for a dynamic panel data model analysis can also be used in Bayesian approach. One of the merits of a Bayesian statistics is that prior information can be included in the analysis. The influence of the prior depends greatly on its precision and on the variance of the variables. Likewise, the data structure has its own influence on the posterior distribution.

The variations within variables are considered and specifically interesting to find appropriate prior information, since not all information can be derived from the data. Therefore, from the Tables it can be seen that for models withnoninformative and informative prior, the numerical standard errors (NSE) are gradually/practically reducing as the precision decreases for noninformative prior and also reduces as the precision value increases at the informative prior. It is noted that the posterior mean of both the noninformative prior and informative prior approached the true value at every level of precision. This is an indication that the data and information about the parameters are very attractive to the posterior distribution.

If the prior variance selected is high, it means researcher is very uncertain about what likely values of parameters are. As a result, the prior precision will be small and little weight will be attached to the parameter prior mean. The posterior mean attaches weight proportional to the precision of prior mean (the inverse of its variance)

Conclusively, Table 1, Table 2 and Table 3 display a better posterior estimate of the parameters of the model at both noninformative and informative hierarchical prior. Hence, Bayesian methods combine data and prior information in a sensible manner to the study of a dynamic individual heterogeneity panel data model.

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