

## CALCULATION OF ONE-NEUTRON REMOVAL CROSS SECTION OF $^{11}\text{Be}$ FROM $^{11}\text{Be} + ^{12}\text{C} \rightarrow ^{12}\text{C} + ^{10}\text{Be} + ^1_0\text{n}$ REACTION SYSTEM

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### Abstract

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*The nucleus of the isotope of Beryllium,  $^{11}_4\text{Be}$ , consisting of four protons and seven neutrons is expected to have a halo structure with the last neutron being loosely bound and having a low binding energy. The removal cross section of such neutron from  $^{10}\text{Be}$  core fragment from  $^{11}\text{Be} + ^{12}\text{C}$  reaction system was computed as a difference between the Total Projectile-Target and Core-Target reaction cross sections in the frame work of the Glauber Theory using the CSC\_GM code. The code was run on Linux operating system. The projectile nucleus is assumed to have the structure of a core plus valence neutron. Both the contributions from the Projectile-Target and Core-Target reaction cross sections are shown. The Projectile-Target reaction cross section and the Core-Target reaction cross-section at energy of 900 MeV per nucleon were found to be 992.4 mb and 852.7 mb respectively. The difference between the two cross sections gave the valence neutron removal cross section as 139.7 mb. The results confirmed the one-neutron halo structure of  $^{11}\text{Be}$ .*

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**Keywords:** *Glauber Model, One-neutron removal, Projectile-target cross sections.*

### 1.0 Introduction

Investigation of the structure of unstable nuclei is one of the current subjects of research today in nuclear physics. Unstable nuclei are considered to have exotic properties such as a halo and skin. “Nuclear skin” is a quantitative measure of the excess of neutrons or protons at the nuclear surface, whereas “nuclear halo” describes such excess plus a tail of the neutron or proton-density distribution [1,2]. Experimental methods and theoretical analysis have been widely used to collect information about unstable nuclei such as; the nuclear size, valence nucleon distribution, skin and halo structure. Details of the structure of unstable nuclei are obtained from the study of nuclear reaction observables such as the total nuclear reaction cross sections, one-nucleon removal cross section, differential elastic cross sections etc. Different approaches had been used to study these reaction observables, which include Adiabatic and Glauber Model [2].

The most promising and widely used model at high projectile energies is the Glauber Model [3]. The model, which is semi-classical and based on the eikonal approximation, describes the nucleus-nucleus interaction in terms of interaction between the constituent nucleons with a given density distribution [4]. It pictures the nuclear collision of high-energy in the impact parameter representation where the nuclei move along the collision direction in a straight path.

The Total reaction cross-section is one of the most fundamental quantities characterizing the nuclear reactions and a probe for nuclear structure details.

In this paper total reaction cross section will be used to obtain the one-nucleon removal cross section of  $^{11}\text{Be}$  core fragment from the  $^{11}\text{Be} + ^{12}\text{C}$  reaction system in the framework of the Glauber Theory using CSC\_GM (Cross Section Calculation in the Glauber model) program code. This will be done by calculating both projectile-target and core-target total reaction cross section of the nucleus. The projectile nucleus  $^{11}\text{Be}$  is assumed to have the structure of a core nucleus ( $^{10}\text{Be}$ ) plus a valence neutron.  $^{11}\text{Be} + ^{12}\text{C} \rightarrow ^{12}\text{C} + ^{10}\text{Be} + ^1_0\text{n}$  describes the reaction system.

Section 2 of the paper contains a brief description of Glauber formalism and calculations. Section 3 describes the methodology. Section 4 contains the results and discussion, and finally, conclusions are described in the section 5.

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**2.0 Theoretical Background**

In a reaction system for the removal of a nucleon from the projectile nucleus the latter is assumed to be a core plus one valence nucleon system, in which the valence nucleon is described with a pure configuration. The projectile is assumed to have no particle bound states except for its ground state.

The projectile nucleus P in the ground state, described with an intrinsic wave function  $\psi_0$ , impinges with momentum  $\hbar K = (0, 0, \hbar K)$  on with a target nucleus T at the initial stage of the reaction. The target nucleus is described with an intrinsic wave function  $\theta_0$  in its ground state. The center-of-mass wavefunction is obtained from  $\psi_0(\theta_0)$ . At the final stage of the reaction, the projectile and the target nucleimove to the states  $a$  and  $c$  specified by a wave functions  $\psi_a$  and  $\theta_c$  respectively. The state  $a$  may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is  $\hbar q$ . The scattering amplitude for this reaction is written in the Glauber theory as an integral over the impact parameter  $b$  between the projectile and the target [5,6,7] as

$$F_{ac}(q) = \frac{i\kappa}{2\pi} \int d\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{b}} \langle \psi_a \theta_c | 1 - \prod_{i \in P} \prod_{j \in T} (1 - \Gamma_{ij}) | \psi_0 \theta_0 \rangle \tag{1}$$

The profile function  $\Gamma$  in eq. (1) is determined so as to fit empirical nucleon-nucleon scattering amplitudes and is taken as follows

$$\Gamma(\mathbf{b}) = \frac{1 - i\alpha}{4\pi\beta} \sigma_{NN} e^{-b^2/2\beta} \tag{2}$$

The argument of  $\Gamma_{ij}$  is  $\mathbf{b} + \mathbf{s}_i^P - \mathbf{s}_j^T$  which stands for the impact parameter between  $i$ th and  $j$ th nucleons.  $\mathbf{s}_i^P$  ( $\mathbf{s}_i^T$ ) is the two-dimensional coordinates comprising the  $x$ - and  $y$ -components of the  $i$ th nucleon coordinate in the projectile (target) relative to its center-of-mass coordinate.

The integrated cross section for this reaction is given by

$$\sigma_{ac} = \int \frac{dq}{\kappa^2} |F_{ac}(q)|^2 \tag{3}$$

The parameters  $\sigma_{NN}$ ,  $\alpha$  and  $\beta$  usually depend on either the proton–proton (neutron–neutron) or proton–neutron case.

**2.1 Total Reaction Cross Section**

The total reaction cross section is obtained by summing  $\sigma_{ac}$  over all possible final states  $ac$  except for the elastic Channel [7] and is given as

$$\sigma_{reac}(P + T) = \sum_{ac \neq 00} \sigma_{ac}, \tag{4}$$

The projectile-target total cross section is given by

$$\sigma_{reac}(P + T) = \int db \left( 1 - |e^{i\chi_{PT}(b)}|^2 \right). \tag{5}$$

Since the projectile is assumed to be a system of a core nucleus coupled with a valence nucleon. Its ground-state wave function is given by

$$\Psi_0 = \phi_0 \Phi_0, \tag{6}$$

where  $\Phi_0$  is the core nucleus and  $\phi_0$  is the halo neutron wave function in the ground state of a halo nucleus ( $\Psi_0$ ).

In the CSC\_GM code, however, the (Optical Limit Approximation) OLA was used for the integration involving the coordinates of the core and the target, while the integration for the valence nucleon coordinate was perform without any approximation [7, 8]. In this treatment, the phase-shift function  $\chi_{PT}$  is given by

$$e^{i\chi_{PT}(b)} = \langle \phi_0 | e^{i\chi_{CT}(b_C) + i\chi_{NT}(b_C + s)} | \phi_0 \rangle, \tag{7}$$

with  $b_C = b - \frac{1}{A_p} \mathbf{s}$ ,  $A_p$  is the projectile mass number,  $\chi_{CT}$  is the core-target phase-shift function,  $\chi_{NT}$  is the nucleon-target phase-shift function and  $\mathbf{s}$  is the two-dimensional coordinates comprising the  $x$ - and  $y$ -components

Also the core-target total reaction cross section is given by

$$\sigma_{reac}(C + T) = \int db \left( 1 - |e^{i\chi_{CT}(b)}|^2 \right). \tag{8}$$

Therefore the one-nucleon removal cross section for halo nuclei can be obtained by approximately by taking the difference between the Projectile-Target and the Core-Target reaction cross sections.

$$\sigma_{reac}(P + T) - \sigma_{reac}(C + T) \tag{9}$$

**3.0 Methodology**

The CSC\_GM code is a Fortran 90 program that was originally run on UNIX operating system. The program is made up of three categories of files: physics source code, input files and output files. The physics source code is the main source code which contains the routine for the actual computations. The input files contain data to be read into the main program at run-time. The output files keep the results of the computation. The program code (CSC\_GM) was first compiled using ‘gnu

compiler' in the digital Linux operating system (Ubuntu 14.04). The compiled program was then tested using the example input files enclosed in the code and it runs interactively with the result obtained found to be as expected. Lastly the compiled program was used in carrying out this work and the data generated were saved in the output files which were later imported into the graphics software Origin50for plotting.

The input data specify the reaction system, the type of cross section, the target and core densities, and the control parameters of the Metropolis algorithm etc. The input data for the <sup>11</sup>Be + <sup>12</sup>C reaction system are read from an input file csc.inp. The configuration of the csc.inp file is as follows: The first line gives the mass numbers of the target, projectile and core (A<sub>T</sub>, A<sub>P</sub>, and A<sub>C</sub>), the second line gives the charge numbers of those nuclei (Z<sub>T</sub>, Z<sub>P</sub>, and Z<sub>C</sub>). This code assumes, A<sub>P</sub> - A<sub>C</sub> = 1. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line define the parameters of the nucleon-nucleon profile function, σ<sub>NN</sub> (in fm<sup>2</sup>), α, and β (in fm<sup>2</sup>). The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line specifies the condition for the Monte Carlo quadrature, the number of configuration points (N<sub>S</sub>), the step size (δ in fm) for the random walk in the Metropolis algorithm, and the seed number for generating random numbers (irand). The seventh line is the control parameter for the single-particle wave function of the valence nucleon. The eighth line gives the number of Gaussians used to fit the target and the core densities. The ninth lines give the coefficients c<sub>i</sub> and the ranges a<sub>i</sub> of the Target density. The last line gives the coefficients c<sub>i</sub> and the ranges a<sub>i</sub> of the core density (in fm<sup>-2</sup>). Results of the computations are written on a file *react.out*. The multi-dimensional integration over the valence nucleon coordinates is performed with a Monte Carlo technique. In the Monte Carlo integration, a set of configuration or integration points is generated according to a suitably chosen guiding function w(x). The random walk with the Metropolis algorithm is taken to generate such points. The code generates the single particle wave function from the parameters. The radial part of the single-particle wave function R<sub>nlj</sub>(r) = ru<sub>nlj</sub>(r), which is used as the guiding function w(x) is obtained by solving a Schrödinger equation,

$$\frac{d^2R(r)}{dr^2} + \frac{2\mu}{\hbar^2} \left[ E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2} \right] R(r) = 0, \tag{10}$$

with a potential U(r) given by

$$U(r) = -V_0 f(r) + V_{ls} (l.s) r_0^{-2} \frac{1}{r} \frac{d}{dr} f(r) + V_{coul}, \tag{11}$$

where

$$f(r) = \left[ 1 + \exp \frac{r - R}{a} \right]^{-1}$$

with

$$R = r_0 A_C^{1/3}.$$

r<sub>0</sub> and a are the radius and diffuseness parameters in fm respectively. V<sub>0</sub> is the initial depth of the potential. V<sub>ls</sub> = 17 MeV [9,10].

**4.0 Results and Discussion**

The <sup>11</sup>Be nucleus is described with a <sup>10</sup>Be + neutron system. The input parameters which have to be filled in the files wf.inp and csc.inp are shown in Table 1 and 2. The one-nucleon removal cross section, obtained using the expressions in Eq. (4) to (9) are written on the output file, *react.out*, as in Table 3. The energy dependence of the projectile-target (P+T) and core-target (C+T) reaction cross section for the reaction system <sup>11</sup>Be + <sup>12</sup>C at different energy per nucleon is shown in Fig.1.

Table 1: wf.inp input file for <sup>11</sup>Be + <sup>12</sup>C reaction system.

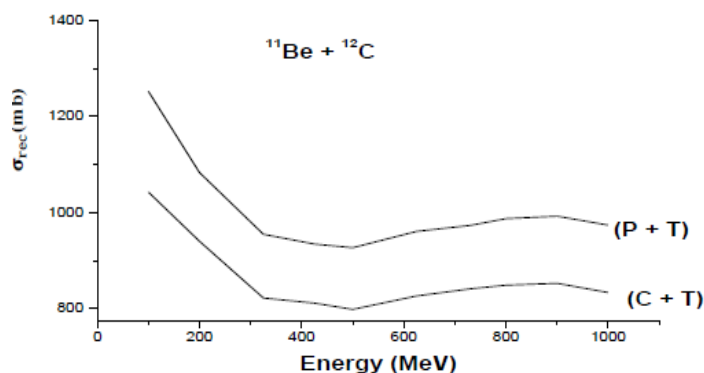
S/N	INPUT PARAMETERS	VALUES
1	Initial depth V <sub>0</sub> of the optical potential (in MeV)	70.0
2	Diffuseness parameter (in fm) a	0.6
3	Radius parameter r <sub>0</sub> (in fm)	1.2
4	Energy eigenvalue for the valence nucleon (in MeV)	-4.946
5	j value for the valence nucleon orbit	0.5
6	Node number for the valence nucleon orbit	1

Table 2: csc.inp input file for <sup>11</sup>Be + <sup>12</sup>C reaction system.

S/N	INPUT PARAMETERS	VALUES
1	Mass numbers of target, projectile and core: (A <sub>T</sub> ; A <sub>P</sub> ; A <sub>C</sub> )	12, 11, 10
2	Atomic numbers of target, projectile and core: (Z <sub>T</sub> ; Z <sub>P</sub> ; Z <sub>C</sub> )	6, 4, 4
3	Incident Energy per nucleon (in MeV)	900
4	Profile function parameters (σ <sub>NN</sub> , α and β in fm <sup>2</sup> )	4.17, -0.217, 0.200
5	l (angular momentum quantum number)	0
6	Monte Carlo parameters (N <sub>s</sub> , δ, irand)	500000, 2.5, -11213
7	icondl(1:available, 0: not available )	0
8	Number of Gaussians used to fit the core and target densities	2
9	Coefficient c <sub>i</sub> , range a <sub>i</sub> (in fm <sup>-2</sup> ) of the Target	-1.25421 0.39907 1.39728 0.355553
10	Coefficient c <sub>i</sub> range a <sub>i</sub> (in fm <sup>-2</sup> ) of the Core	-1.19721 0.300572 1.35334 0.255236

Table 3. The sample output for  $^{11}\text{Be} + ^{12}\text{C}$  reaction system at 900 MeV.

S/no	Cross Section (in mb)	Result
1	Projectile reaction cross section	992.4
2	Core reaction cross section	852.7
3	Nucleon removal cross section	139.7

Figure 1: Energy dependence of (P + T) and (C + T) reaction cross sections for  $^{11}\text{Be} + ^{12}\text{C}$  system

### Conclusion

The CSC\_GM program was used to compute the one-nucleon removal cross section from total reaction cross sections of  $^{11}\text{Be}$  core fragment from  $^{11}\text{Be} + ^{12}\text{C}$  reaction system in the framework of the Glauber theory. The ground state of  $^{11}\text{Be}$  is assumed to be  $^{11}\text{Be} + \text{neutron}$  system. The valence neutron is in the  $1s_{1/2}$  orbit with  $n = 1$ ,  $l = 0$ ,  $j = 0.5$ , and the separation energy  $\varepsilon = 0.22$  MeV [11,12]. The one-nucleon removal cross section of  $^{11}\text{Be} + ^{12}\text{C}$  in this case was obtained by the difference between the projectile-target (P+T) and core-Target (C+T) reaction cross section. The gap between (P+T) and (C+T) is large (Fig.1), reflecting the halo structure in  $^{11}\text{Be}$ . The higher values of these cross sections obtained, correspond to the low binding energy for last the last neutron in  $^{11}\text{Be}$  nucleus. The wider gap between (P+T) and (C+T) occurs in the energy region beyond a few hundred MeV per nucleon; this indicated that Glauber model is appropriate at high energy, at low energy however the assumptions are poor. This is because at low energy the Glauber scattering amplitude reduces to Born approximation.

### References

- [1] Ozawa, A., Suzuki, T., and Tanihata, I. (2001). Nuclear Size And Related Topics, Nuclear Physics A 693 Pp.32–62.
- [2] Al-Khalili, J., and Nunes, F. (2003). Reaction Models to Probe The Structure of Light Exotic Nuclei. Journal of Physics. G: Nucl. Part. Phys. **29**, (R89–R132), pp.93–98.
- [3] Khoa D. T., Hoang S. T. and Marcella G. (2003) Microscopic Study of Interaction Cross Sections Measured at Relativistic Energies For Stable And Unstable Nuclei. Nuclear Physics A 722 Pp.92
- [4] Esha, R. (2012). Glauber Modeling of High Energy. National Institute of Science Education and Research Bhubaneswar. pp.1–2.
- [5] Adamu, I. D. (2013). Calculation Of Momentum Distributions Of 7 Be Fragment From  $^8\text{B} + ^{12}\text{C}$  Reaction Using The Glauber Theory. IJRRAS17 (1), pp.116-121.
- [6] Tanihata, I. (1995). Nuclear Structure Studies From Reaction Induced By Radioactive Nuclear Beams. Progress in Particle and Nuclear Physics. Vol. 35, Pp. 505-573.
- [7] Ogawa, Y., Suzuki, Y., and Tanihata, I. (2003). Cross Section Calculations In Glauber Model : I . Core Plus One-Nucleon Case ☆, 151, 369–386. [http://doi.org/10.1016/S0010-4655\(02\)00734-8](http://doi.org/10.1016/S0010-4655(02)00734-8)
- [8] Abu-Ibrahim, B., and Suzuki, Y. (2002). Calculation of Nucleus-Nucleus Cross sections at Intermediate Energies Using Glauber Theory. Nuclear physics A 706, 111-122.
- [9] Sharma, Mahesh K. Sharma And Manoj K. R. N. Panda, S. K. P. (2015). Quest Of  $^{37}\text{Mg}$  Halo Structure Using Glauber Model And Microscopic Relativistic Mean Field Densities, APS Journals -American Physical Society 58. Pp.461.
- [10] Shukla, A., Sharma, B. K., Chandra, R., Arumugam, P., & Patra, S. K. (2014). Nuclear Reaction Studies Of Unstable Nuclei Using Relativistic Mean Field Formalisms In Conjunction With Glauber Model, APS Journals, 6,: 21.10.-k, 21.10.dr, 21.10.Ft, 21.30.-x, 24.10.-1, 24.10.Jv.
- [11] Glauber, R.J., Brittin, W.E., and Dunham (Eds.) L.C., Lecture on Theoretical Physics, Vol. 1, Interscience, New York, 1959, p.315.
- [12] Sharma, M. K., Panda, R. N., Sharma, M. K., & Patra, S. K. (2013). Reaction And Structure Effects Of Light Mass Nuclei Using Glauber Model With Relativistic And Non Relativistic Effective Interaction Densities. Journal of American Physical Society 25. Pp. 8–9.